
Index

- adjacency matrix, 104
- adjoint representation, 159
- adjugate matrix, 168
- affine cone, 28
- affine variety, 19, 117
- algebraic statistics, 22
- algebraic torus, 110, 115, 158
- analytic center, 198
- Aronhold invariant, 145, 168, 180
- Artin's Theorem, 93
- assignment problem, 105
- associated prime, 47

- Bertini's Theorem, 27
- Bézout's Theorem, 26, 80
- binomial, 118
- Birch point, 126
- border rank, 142

- Catalan number, 84
- character, 116, 156–158
- Chevalley's Theorem, 30
- Chinese Remainder Theorem, 23
- Cholesky factorization, 197, 200
- class function, 157
- closed image, 58, 61, 117
- complementary slackness, 193
- complete intersection, 26
- complete symmetric polynomial, 159
- cone, 120
- conic curve, 32
- constructible set, 25, 68
- coordinate ring, 22

- cosine moment curve, 194
- covariance matrix, 134
- cubic curve, 32
- cusp, 35, 180
- cuspidal curve, 35, 117

- degree, 2, 14, 16, 25
 - of a Grassmannian, 84
- Derksen's algorithm, 181
- Descartes' rule of signs, 40
- Dickson's Lemma, 7
- dimension, 14, 16, 25
 - of a face, 120
- discriminant, 178, 181, 183
- dual curve, 191

- Ehrhart
 - polynomial, 213
 - series, 211
- eigenspace
 - of a tropical matrix, 108
- eigenvalue, 108, 193
- eigenvector
 - of a symmetric matrix, 134
 - of a tensor, 138
- elementary symmetric polynomial, 159, 172
- elimination ideal, 58, 60, 61, 63
- elliptic curve, 32, 180
- embedded prime, 47
- Erlanger Programm, 171
- Euclidean algorithm, 4, 6
- exponent of matrix multiplication, 148

- extended Buchberger algorithm, 89
- extended Euclidean algorithm, 6
- f-vector, 120, 127
- face, 120
- facet, 120
- Fano plane, 204
- Farkas' Lemma, 94
- flag of linear subspaces, 80
- flattening of a tensor, 141, 164
- Fundamental Theorem of Algebra, 25
- Fundamental Theorem of Tropical Geometry, 111
- Gaussian graphical model, 198
- Grassmannian, 62, 73, 75, 162, 178, 208, 209
- Gröbner basis, 9, 60, 78, 88
- hexagon invariant, 182
- highest weight, 159
- Hilbert function, 12, 15, 211, 213
- Hilbert polynomial, 13, 14, 16
- Hilbert series, 12, 16, 175, 211
 - multigraded, 212
- Hilbert's Basis Theorem, 9, 42
- Hilbert's Finiteness Theorem, 174
- Hilbert's Nullstellensatz, 78, 90
- homogeneous ideal, 28
- homogeneous polynomial, 28
- homogeneous variety, 75, 162
- hyperdeterminant, 62, 63, 72, 142, 182
- hypersimplex, 209
- ideal, 3
- ideal quotient, 4, 48
- implicitization, 61, 118
- independence model, 21, 125
- initial
 - form, 110
 - ideal, 9, 214
 - monomial, 8
- integer decomposition property, 123
- integral domain, 5
- invariant ring, 172, 175
- irreducible
 - ideal, 44
 - representation, 153, 160
 - variety, 20–22
- isotypic component, 156
- j-invariant, 180
- Jacobian matrix, 27
- Kalman variety, 30
- Kapranov's Theorem, 110, 111
- KKT equations, 193, 195, 200
- Kleene plus, 107
- Kostant's Theorem, 169
- Lagrangian Grassmannian, 85
- Lasserre hierarchy, 198
- lattice, 116
- lattice polytope, 121, 206, 211
- Laurent monomial, 116
- Laurent polynomial, 110, 111
- lexicographic order, 8, 12
- linear programming duality, 94
- Littlewood-Richardson rule, 83
- log-linear model, 126
- Macaulay2, 78
- Main Theorem of Elimination Theory, 70
- Maschke's Theorem, 154
- matrix completion, 59, 198
- matrix multiplication tensor, 146, 150
- matroid, 202
 - algebraic, 202
 - basis of, 203
 - basis polytope, 206, 209
 - graphic, 202
 - rank, 203
 - representable, 202
 - restriction, 207
 - uniform, 202
 - union, 207
- maximal ideal, 5
- maximum likelihood, 125, 126, 128, 198
- minimal prime, 47, 48, 216
- Möbius strip, 32
- modular invariant theory, 174
- Molien's Theorem, 175, 177
- moment map, 124, 131
- monomial ideal, 9, 49, 214
- monomial order, 8
- morphism
 - of representations, 153
- Motzkin polynomial, 93, 96, 196
- multigraded Hilbert series, 212
- Newton's identities, 172
- nilpotent
 - matrix, 91
 - ring element, 5
- node, 180

- Noether's degree bound, 175
 Noetherian ring, 44
 nonnegative rank, 145
 normal polytope, 130, 206
 normal variety, 118, 121, 122
 normalized volume, 215
 nullcone, 179
 Nullstellensatz, 87
 certificate, 89

 odeco, 139
 orbit, 162
 orthogonal decomposition, 136, 139
 oval, 32

 PDE, 49
 Pfaffian, 62, 78, 168
 PID, 4
 Pieri's rule, 83
 plethysm, 167
 Plücker quadric, 77, 168
 Plücker relations, 62, 76, 78, 178
 polytope, 121, 206
 positive definite, 188
 positive semidefinite, 188
 positive toric variety, 124
 Positivstellensatz, 95
 power iteration, 138
 power sum polynomial, 172
 primary decomposition, 45
 primary ideal, 5, 44, 45, 216
 prime ideal, 5, 21, 22, 43
 projective closure, 28
 projective space, 27, 31
 projective variety, 27, 28, 75, 121, 214
 PSD, 188
 pseudoline, 32
 Puiseux series, 1, 40, 101, 109

 quadratic module, 95
 quotient space, 173

 radical ideal, 5, 91
 radical of an ideal, 17, 90
 rank
 of a tensor, 137
 rational map, 134
 rational normal curve, 29
 real Nullstellensatz, 92, 95, 187
 real radical, 95
 realization space, 204
 reduced Gröbner basis, 9
 reductive group, 155

 representation, 152
 completely reducible, 155
 semi-simple, 155
 resultant, 63
 Reynolds operator, 173
 Riemann sphere, 31
 Robinson polynomial, 97
 roots
 in representation theory, 159

 Salmon problem, 169
 saturated monoid, 118
 saturation, 48
 scheme, 54
 Schubert calculus, 79, 81
 Schubert variety, 80
 Schur polynomial, 161
 Schur's Lemma, 154
 Schur-Weyl duality, 162
 SDP, 191
 secant variety, 144, 166
 Segre variety, 122, 141
 semi-simple
 representation, 155
 semialgebraic set, 25, 67
 semiring, 100
 simplicial complex, 202
 singular locus, 26, 27
 singular point, 26
 singular value decomposition, 136
 singular vector, 135, 140
 smooth point, 27
 SOS programming, 198
 Specht module, 163
 spectrahedral shadow, 194
 spectrahedron, 188
 spectral decomposition, 136
 Spectral Theorem, 187
 spectrum of a ring, 22
 standard monomial, 10
 standard Young tableau, 83
 Strassen equation, 168
 Strassen invariant, 145
 Strassen's algorithm, 147
 submodularity, 203
 subrepresentation, 153
 sum of squares, 92
 supporting hyperplane
 of a cone, 120
 Sylvester matrix, 65
 symmetric exchange property, 203
 symmetric polynomials, 172

- symmetric rank, 144
- tangent space, 26
- tangential variety, 143
- Tarski-Seidenberg Theorem, 67
- tensor decomposition, 137
- Toeplitz matrix, 194
- Toeplitz spectrahedron, 194
- toric ideal, 118
- toric model, 126
- toric variety, 117, 121, 128, 209, 214
 - normal, 118
- torus orbit, 123
- transportation polytopes, 125
- triangular prism, 124
- triangulation, 211, 214, 216
- tropical
 - convexity, 109
 - determinant, 105
 - eigenvalue and eigenvector, 106
 - linear space, 205
 - polynomial, 102, 110
 - polytope, 109
 - projective torus, 108
 - rank, 104, 106, 111
 - root, 102
 - semiring, 99
 - triangle, 109
 - variety, 111
 - zero, 110
- tropicalization
 - of a polynomial, 102, 110
 - of a variety, 111
 - of the determinant, 105
- ultrametric, 101
- unstable points, 179

- valuation, 101
- value group, 101
- variety, 19, 22
- Veronese variety, 71, 122, 162
- vertex of a polytope, 206

- W -state, 142, 149
- Weak Duality Theorem, 192
- Weierstrass form, 32
- weight spaces, 158
- weights, 158

- Young diagram, 84, 160
- Young flattening, 166

- Zariski topology, 20, 22
- zero divisor, 5