

---

# Contents

<b>Preface</b>	ix
A Note for the Student	x
<b>Chapter 1. Plane Curves</b>	1
§1.1. The Affine Plane	1
§1.2. The Projective Plane	4
§1.3. Plane Projective Curves	8
§1.4. Tangent Lines	14
§1.5. The Dual Curve	19
§1.6. Resultants and Discriminants	25
§1.7. Nodes and Cusps	30
§1.8. Hensel's Lemma	39
§1.9. Bézout's Theorem	44
§1.10. The Plücker Formulas	50
Exercises	51
<b>Chapter 2. Affine Algebraic Geometry</b>	55
§2.1. The Zariski Topology	55
§2.2. Some Affine Varieties	62
§2.3. Hilbert's Nullstellensatz	63
§2.4. The Spectrum	67
§2.5. Morphisms of Affine Varieties	71
§2.6. Localization	76
§2.7. Finite Group Actions	80
Exercises	85

---

<b>Chapter 3. Projective Algebraic Geometry</b>	89
§3.1. Projective Varieties	90
§3.2. Homogeneous Ideals	93
§3.3. Product Varieties	97
§3.4. Rational Functions	101
§3.5. Morphisms	104
§3.6. Affine Varieties	111
§3.7. Lines in Three-Space	113
Exercises	120
<b>Chapter 4. Integral Morphisms</b>	125
§4.1. The Nakayama Lemma	125
§4.2. Integral Extensions	127
§4.3. Normalization	130
§4.4. Geometry of Integral Morphisms	135
§4.5. Dimension	138
§4.6. Chevalley's Finiteness Theorem	143
§4.7. Double Planes	149
Exercises	155
<b>Chapter 5. Structure of Varieties in the Zariski Topology</b>	159
§5.1. Local Rings	159
§5.2. Smooth Curves	164
§5.3. Constructible Sets	170
§5.4. Closed Sets	173
§5.5. Projective Varieties Are Proper	175
§5.6. Fibre Dimension	178
Exercises	179
<b>Chapter 6. Modules</b>	183
§6.1. The Structure Sheaf	183
§6.2. $\mathcal{O}$ -Modules	184
§6.3. The Sheaf Property	187
§6.4. More Modules	190
§6.5. Direct Image	198
§6.6. Support	202

---

§6.7. Twisting	203
§6.8. Extending a Module: Proof	209
Exercises	213
<b>Chapter 7. Cohomology</b>	217
§7.1. Cohomology	217
§7.2. Complexes	219
§7.3. Characteristic Properties	222
§7.4. Existence of Cohomology	223
§7.5. Cohomology of the Twisting Modules	230
§7.6. Cohomology of Hypersurfaces	232
§7.7. Three Theorems about Cohomology	234
§7.8. Bézout's Theorem	239
§7.9. Uniqueness of the Coboundary Maps	241
Exercises	243
<b>Chapter 8. The Riemann-Roch Theorem for Curves</b>	247
§8.1. Divisors	247
§8.2. The Riemann-Roch Theorem I	253
§8.3. The Birkhoff-Grothendieck Theorem	256
§8.4. Differentials	258
§8.5. Branched Coverings	262
§8.6. Trace of a Differential	264
§8.7. The Riemann-Roch Theorem II	271
§8.8. Using Riemann-Roch	273
§8.9. What Is Next	283
Exercises	284
<b>Chapter 9. Background</b>	289
§9.1. Rings and Modules	289
§9.2. The Implicit Function Theorem	302
§9.3. Transcendence Degree	303
<b>Glossary</b>	305
<b>Index of Notation</b>	311
<b>Bibliography</b>	313
<b>Index</b>	315