
Notation Index

- $\#A$, the number of elements of set A , 6
 \ll , absolute continuity of one measure with respect to another, 162
 \perp , mutual singularity of measures, 162
 $\|\cdot\|_{L^1_{\text{loc}, \text{unif}}}$, 433
 $\|k\| = \max(1, |k|)$, 364
- $\text{AC}([a, b])$, set of absolutely continuous functions on $[a, b]$, 247
 $\text{AC}^2_{\text{loc}}(I) = \{f \in \text{AC}_{\text{loc}}(I) \mid f' \in \text{AC}_{\text{loc}}(I)\}$, 379
 $\text{AC}_{\text{loc}}(I)$, set of locally absolutely continuous functions on I , 250
 $A^c = X \setminus A$, the complement of A in space X , 2
- \mathcal{B}_X , the Borel σ -algebra on X , 3
 $\mathcal{B}_b(X)$, the algebra of bounded Borel functions from X to \mathbb{C} , 34
- $C(K) = C(K, \mathbb{C})$, the space of continuous maps $K \rightarrow \mathbb{C}$, 48
 $C(K, \mathbb{R})$, the space of continuous maps $K \rightarrow \mathbb{R}$, 50
 $C_0(\mathbb{R})$, the set of continuous decaying functions on \mathbb{R} , 195
 $C_A(\psi)$, cyclic subspace of vector ψ , 141
 $\hat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$, the Riemann sphere, 184
 \mathbb{C}_+ , the upper half-plane, 183
 $\mathbb{C}[x]$, algebra of polynomials with complex coefficients, 118
- \mathbb{D} , the unit disk in \mathbb{C} , 184
- $D(A)$, domain of unbounded operator A , 227
 δ_x , the Dirac measure at x , 6
 Δ , discriminant of a periodic Schrödinger operator, 444
 $D(x, z)$, Weyl disk, 409
- \mathcal{F} , Fourier transform on $L^2(\mathbb{R})$, 292
 f_A , Möbius transformation induced by matrix A , 184
 \hat{f} , eigenfunction expansion of f , 400, 427
 \hat{f} , Fourier transform of f , 291
- \check{g} , inverse eigenfunction expansion of g , 400, 427
 \check{g} , inverse Fourier transform of g , 291
 $G(x, y; z)$, Green's function, 391
- \mathcal{H}_{ac} , the absolutely continuous subspace for A , 274
 $\mathcal{H}_{\alpha\text{c}}$, the α -continuous subspace for A , 275
 $\mathcal{H}_{\alpha\text{s}}$, the α -singular subspace for A , 275
 $\mathcal{H}_{\text{cont}}$, the continuous subspace for A , 273
 h_{\pm} , the positive and negative parts of a function, 21
 \mathcal{H}_{pp} , the pure point subspace for A , 272
 \mathcal{H}_{sc} , the singular continuous subspace for A , 274
 \mathcal{H}_{ss} , the singular subspace for A , 274
- Ker , the kernel of an operator, 60

- $\mathcal{L}(X, Y)$, the set of bounded linear operators from X to Y , 59
 $L^p(X, \mu)$, 54
 $L^p([a, b])$, L^p space on $[a, b]$ with respect to Lebesgue measure, 247
 $\ell^p(X)$, L^p -space with counting measure on X , 58
 $L_c^p(I)$, set of compactly supported functions in $L^p(I)$, 400
 $L_{\text{loc}}^p(I)$, set of locally L^p functions on I , 250

 $m(x, z)$, 421
 $m_{\pm}(x, z)$, 422
 μ_{α} , a Lebesgue–Stieltjes measure, 26
 $\mu \otimes \nu$, product measure, 31
 $M(z)$, Weyl M -matrix, 219, 326, 426

 $\tilde{o}(\)$ asymptotic notation, 415

 $\mathcal{P}(X)$, the set of subsets of X , 1
 $\psi_{\pm}^{\pm}(x) = \psi^{\pm}(x, z)$, Weyl solution, 390

 Ran , the range of an operator, 60
 $\text{Ran}_{\mu} g$, essential range of g with respect to μ , 144
 $\hat{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$, the extended real line, 13

 $\Theta(z)$, Marchenko–Ostrovski map, 338, 445

 \mathcal{T}_X , the metric topology on metric space X , 3

 $W(f, g)$, Wronskian, 381
 $W_{\pm}(f, g)$, endpoint Wronskians, 381

 X_{\pm} , endpoint domains for Schrödinger operators, 380
 X_{\pm}^* , null subspaces of endpoint domains, 381

 Y_{\pm} , endpoint domains for self-adjoint Schrödinger operators, 388

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- absolutely continuous spectrum, 274
- adjoint
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 - of direct sum of operators, 120
 - of matrix, 108
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