
Notation index

- $\Delta(\alpha; r)$, 393
 $\Pi_{\mathcal{V}}$, 130
 $\dot{+}$, 43
 \oplus , 126
 α_j , 37
 γ_j , 36
 $\sigma(A)$, 37
- $(A; \mathcal{X})_{\max}$, 305
 $(A; \mathcal{X})_{\min}$, 305
 $A \succ B$, 171
 $A \succeq B$, 171
 A^* , 118
 A^H , 4
 A^T , 4
 $A^{1/2}$, 179
 A^\dagger , 164
 $A_{[j,k]}$, 173
 $A_{\{I;J\}}$, 194
 $A_{\{i,j,k;r,s,t\}}$, 197
 $A_{\{i;j\}}$, 69
 $\|A\|_s$, 101
 $\|A\|_{s,t}$, 102
 \mathbb{A} , 330
 $|A|$, 65
- $C_\mu^{(p)}$, 40
 \mathbb{C} , 1
 $\mathbb{C}^{p \times q}$, 1
 \mathbb{C}^p , 1
 \mathbb{C}_L , 403
 \mathbb{C}_R , 403
 $\mathcal{C}(Q)$, 235
- $\mathcal{C}^k(Q)$, 88, 235
 $(D_{\mathbf{u}}f)$, 263
 $\det A$, 65
 $\dim \mathcal{V}$, 3
 $\mathcal{E}_+(A)$, 313, 409
 $\mathcal{E}_-(A)$, 313, 409
 $\mathcal{E}_0(A)$, 313, 409
 (∇f) , 237
 $G_{[k,m]}$, 467
 $H_f(\mathbf{x})$, 237
 \mathcal{H}_α , 465
 \mathcal{H}_\bullet , 466
 I_n , 3
 $I_{\mathcal{U}}$, 7
 $\int_{\Gamma} f(\lambda) d\lambda$, 382
 $J_{\mathfrak{f}}$, 239
 $K_\omega^{(\alpha)}$, 465
 $K_\omega^{(\bullet)}$, 466
 \mathcal{N}_T , 7
 \mathcal{N}_A , 7
 $O_{p \times q}$, 3
 P_Γ^A , 386
 $P_{\mathcal{V}}^{\mathcal{W}}$, 128
 R_α , 464

R_{\bullet} , 467 \mathbb{R} , 1 $\mathbb{R}^{p \times q}$, 1 \mathbb{R}^p , 1 $\mathbb{R}_{>}^{p \times q}$, 281 $\mathbb{R}_{\geq}^{p \times q}$, 281 \mathcal{R}_T , 7 \mathcal{R}_A , 7 $r_{\sigma}(A)$, 120 $\mathcal{S}_{\mathbf{b}}$, 359 T^* , 118 $T_k(x)$, 376 $\|T\|_{\mathcal{U}, \mathcal{V}}$, 103 $\langle \mathbf{u}, \mathbf{v} \rangle_A$, 371 $\langle \mathbf{u}, \mathbf{v} \rangle_{\mathcal{U}}$, 113 \mathcal{V}^{\perp} , 126 $W(A)$, 429 \mathcal{X}' , 345 $\|\mathbf{x}\|_s$, 100 $\langle \mathbf{x}, \mathbf{y} \rangle_{\text{st}}$, 114 \mathcal{Z}_{Ω} , 413

Subject index

- adjoint, 118
- Aiken, John G., 344
- Akritis, Alkiviadis G., 198
- Akritis, Evgenia K., 198
- algebraic multiplicity, 37
- algorithm, 231
- analytic, 381
- Ando, Tsuyoshi, 323
- Andrica, Dorin, 97
- angle between subspaces, 154
- Apostol, Tom M., 235, 236
- approximate solutions, 169
- approximating by unitary matrices, 337
- area of parallelogram, 156
- arithmetic mean, 90
- Arov, Damir Z., 474

- Banach space, 111
- Barnett identity, 455
- basis, 3
- Bau, III, David, 379
- Belevich, Vitold, 402
- Bessel's inequality, 131
- best approximation, 166, 167
- Bezoutian, 453
- Bhatia, Rajendra, 303, 314, 320, 323, 460
- Binet-Cauchy formula, 189
- binomial formula, 9
- Birkhoff-von Neumann theorem, 297
- block Gaussian elimination, 32
- block multiplication, 6
- block triangular matrices, 18

- Bollobás, Béla, 97, 357
- Borwein, Jonathan M., 122
- bounded, 349
- Bram, Joseph, 280
- Brouwer fixed point theorem, 246
- Brualdi, Richard A., 198
- Burg, John P., 422

- Carathéodory, Constantin, 96
- Cassels, John W. S., 401
- Cauchy interlacing theorem, 308
- Cauchy sequence, 111
- Cauchy-Riemann equations, 260
- Cauchy-Schwarz inequality, 93, 115, 157
- Cayley-Hamilton theorem, 55, 73
- characteristic polynomial, 56, 72, 76, 188
- Chebyshev polynomial, 376
- circulant, 79
- companion matrix, 75
- complementary space, 45
- completing the square, 200
- complex vector space, 1
- conjugate gradient recursion, 374
- conjugate gradients, 369
- conservation of dimension, 13
- continuity of eigenvalues, 387
- contour integral, 382
- contractive fixed point theorem, 243
- contractive matrix, 208
- controllable pair, 399
- convergence estimates, 375
- convex function, 85, 241, 434

- convex hull, 432
 convex set, 85
 Conway, John B., 357
 cosine, 154
 Courant-Fischer theorem, 305
 Cramer's rule, 71
 CS decomposition, 458
 Curgus, Branko, 460
 cyclic matrix, 138
 cyclic vector, 83
- Davis, Chandler, 428
 de Boor, Carl W. R., 436
 de Branges space, 463
 de Branges, Louis, 473
 de Sá, Eduardo M., 422
 Degani, Ilan, 460
 determinant, 65
 determinant identities, 193
 Deutsch, Emeric, 422
 diagonalizable, 37
 Diepold, Klaus, 160
 difference equations, 214, 217
 differentiability of eigenvalues, 312
 differentiating determinants, 185
 differentiation of matrix-valued functions, 222
 Dijkstra, Aad, 460
 dimension, 3
 direct sum decomposition, 43, 45, 46
 directional derivative, 263
 discrete dynamical system, 211
 doubly stochastic matrix, 297
 Douglas, Ron G., 428
 Doyle, John C., 428, 447
 dual extremal problems, 360
 Duren, Peter L., 249
- Eckhart, Carl, 170
 eigenvalue, 35, 41, 72
 eigenvector, 35, 41
 entire function, 463
 equivalent norms, 100
 Erdos, John A., 344
 exponential of a matrix, 222
 extremal problems, 263, 266
 extreme point, 96, 297
- Fan, Ky, 300, 310
 Farkas's lemma, 353
 Feintuch, Avraham, 428
 Fejér, Leopold, 401
- Fibonacci sequence, 217
 fitting a line, 204, 205
 fixed point, 243
 fixed point theorems, 246
 Fourier matrix, 81
 fractional powers of positive definite matrices, 401, 434
 Frobenius norm, 114, 166, 338
 Fuglede, Bert, 145
 Fujii, J. I., 436
 Fujii, M., 436
 Furuta, T., 436
- Gauss, Carl F., 249
 Gauss-Lucas theorem, 433
 Gauss-Seidel method, 30
 Gaussian elimination, 21
 Gaussian quadrature, 449
 generalized backward shift, 464
 generalized eigenvector, 37
 generalized Vandermonde matrix, 78
 generic, 390
 geometric mean, 90
 geometric multiplicity, 36
 Geršgorin disks, 393
 Glüch, Jochen, 290
 Glazman, Israel N., 160
 Glover, Keith, 428, 447
 Glunt, William, 422
 Gohberg, Israel, 170, 323, 422
 Goldstein, Jerome A., 344
 Golub, Gene H., 170
 Google matrix, 292
 gradient, 237
 Gram matrix, 116, 117, 130
 Gram-Schmidt method, 133
 Grone, Robert, 422
 Gronwall's inequality, 226
- Hadamard's inequality, 191
 Hadamard, Jacques, 145
 Hahn-Banach extension theorem, 349
 Halmos, Paul, 436
 Hamiltonian matrix, 437
 Hankel matrix, 82, 408, 461
 Hardy, Godfrey H., 323, 401
 Hautus, Malo L. J., 402
 Hayden, Thomas L., 422
 Heinz inequality, 435
 Heinz, E., 436
 Helton, J. William, 436
 Hermitian matrix, 137, 140, 144

- Hermitian transpose, 4
Hessian, 237, 241
Heymann, Michael, 402
Hilbert matrix, 400
Hilbert space, 114
Hoffman, Alan, 170
Hölder's inequality, 91
holomorphic, 381
homogeneous system, 211
hyperplane, 348
- identity matrix, 3
implicit function theorem, 255
inequalities for determinants, 191, 192
inertia, 410
inner product, 113
inner product space, 113
integration of matrix-valued functions, 222
invariant subspace, 39, 41
inverse, 6
inverse function theorem, 251
invertible, 6
irreducible matrix, 281, 285
isometric matrix, 137, 161
isospectral, 226
- Jacobi matrix, 308
Jacobi's determinant identity, 195
Jacobi's formula, 187
Jacobi, Carl G., 198
Jacobian, 239
Jensen's inequality, 85
Johnson, Charles R., 422
Jordan cells, 40
Jordan chain, 55, 56
Jordan decomposition, 53
- Kaashoek, Marinus A., 422
Kahan, William M., 428
Kantorovich, Leonid V., 280
Kato, Tosio, 391
keep in mind, 13–15, 28, 65, 122, 137, 140, 162, 168, 176, 182, 185
Krantz, Steven G., 261
Krein, Mark G., 160, 170, 323
Krein-Milman theorem, 96
Krylov subspace, 377
Ky Fan's inequality, 310
Ky Fan's maximum principle, 309
- Lagrange multipliers, 266
Lancaster, Peter, 411, 447
Lax pair, 226
left invertible, 5, 14, 110
Leibniz's rule, 77
Leray-Schauder theorem, 246
Leslie matrices, 294
Levy, Eliahu, 209
Lewis, Adrian S., 122
Li, C-K., 391
Lidskii's inequality, 310
Lidskii, Viktor B., 310
linear combinations, 2
linear dependence, 2
linear functional, 345
linear independence, 2
linear mapping, 7
Lippert, Robert, 314
Littlewood, John E., 323, 401
Ljubic, Ju. L., 160
lower triangular, 8
LQR problem, 445
LU factorization, 174
Luenberger, David G., 379
Lyapunov equation, 407
- Malaschonok, Genadii I., 198
mappings, 6
matrices with nonnegative entries, 281
matrices with positive entries, 281
matrix completion, 413
matrix multiplication, 4
maximum entropy completion, 417
McIntosh, Alan, 436
mean value theorem, 235, 238
Mihaly, Bakonyi, 436
minimal norm completion, 423, 424, 426
minimal polynomial, 55, 73
minimum matrix, 295
Minkowski's inequality, 93
minor, 69
Mirsky, Leonid, 170
moment problem, 422, 461, 474
Moore Penrose inverse, 164
- Nakamoto, R., 436
negative definite, 171
negative semidefinite, 171
Newton recursion, 277
Newton step, 247, 277
Newton's method, 247, 275
Ninio, F., 290
nonhomogeneous system, 214
norm, 99

- normal matrix, 137, 140, 142, 143
 normal transformation, 145
 normed linear space, 99
 notation, 1, 114, 128, 171, 173, 235,
 251, 284, 305, 313, 347, 371, 403,
 418, 467
 nullspace, 7
 numerical range, 429

 observable, 402
 open mapping theorem, 253
 operator norm, 103
 orthogonal, 125
 orthogonal complement, 126, 130
 orthogonal decomposition, 126
 orthogonal family, 125
 orthogonal matrix, 4, 137, 140
 orthogonal projection, 129, 130, 201
 orthonormal expansion, 126
 orthostochastic matrix, 300
 orthonormal family, 126

 parallelogram law, 116
 Parks, Harold R., 261
 Parrott's lemma, 424
 partial isometry, 161, 180
 Peller, Vladimir V., 401
 permutation matrix, 4, 63, 137
 Perron-Frobenius theorem, 285
 pivot column, 22
 pivot variables, 22
 pivots, 22
 Polak, Elijah, 280
 polar form, 180
 polarization identity, 116
 Polya, George, 323, 401
 Popov, Vasilie M., 402
 positive definite, 149, 171, 176
 positive semidefinite, 171
 principal submatrix, 188
 products of singular values, 167
 projection, 127
 projection by iteration, 147
 projection formulas, 151, 153
 Putnam, Calvin R., 145

 QR factorization, 134, 457
 quadrature formulas, 452

 range, 7
 rank, 13, 14
 real Jordan forms, 62
 real vector space, 1

 reproducing kernel, 462
 reproducing kernel Hilbert space, 462
 resultants, 456
 Riccati equation, 437
 Riesz projection, 386
 Riesz, Frigyes, 401
 right invertible, 6, 14, 110
 Rodman, Leiba, 447
 roots of polynomials, 258
 Rudin, Walter, 122

 Saaty, Thomas L., 280
 Santanu, Sartu, 474
 Schneider, Hans, 198, 422
 Schur complements, 34, 200, 207
 Schur's theorem, 141
 Schur, Issai, 141, 145
 selfadjoint transformation, 145
 Shalit, Orr M., 209
 Sherman Morrison formula, 74
 Shewchuk, Jonathan R., 379
 shifting eigenvalues, 400
 Shuchat, Alan, 209
 similar, 37
 Simon, Barry, 97
 simple curve, 382
 simple permutation matrix, 63
 sine, 155
 singular value decomposition, 162, 164
 singular values, 164, 165, 168
 skew-Hermitian matrix, 137
 smooth, 235
 span, 2
 spectral mapping, 74, 398
 spectral radius, 120, 121, 396
 spectrum, 37
 Spitkovsky, Ilya M., 160, 436
 square root, 178, 179
 standard inner product, 114
 Stein equation, 405
 Stewart, G. W., 170
 stochastic matrix, 291
 strictly convex, 241, 370
 strictly convex function, 85, 90, 201, 241
 strictly convex normed linear space, 201
 sublinear functional, 348
 subspace, 1
 sum of subspaces, 43
 sums of singular values, 167
 svd, 164
 Sylvester equation, 405
 Sylvester's determinant identity, 196

- Sylvester's law of inertia, 314
symmetric gauge function, 320
symmetric matrix, 140
- Tao, Terence, 198
Tarazaga, Pablo, 422
Taylor's formula, 236
Tismenetsky, Miron, 411
Toeplitz matrix, 212, 405, 461
Toeplitz-Hausdorff theorem, 430
trace, 73, 90, 114, 158
transpose, 4
Trefethen, Lloyd N., 379
triangle inequality, 99
triangular, 8
triangular factorization, 173
triangular matrices, 18
- UL factorization, 174, 200
unitarily invariant norm, 319
unitary matrix, 137, 140, 161
unitary transformation, 145
upper echelon matrix, 22
upper triangular, 8
- Vandermonde matrix, 78
volume of parallelepiped, 157
von Neumann's inequality, 208
von Neumann's trace inequality, 301
von Neumann, John, 301, 320
- warning, 7, 122, 137, 161, 172, 201, 353
warnings, 264, 265
Watkins, David, 460
Webster, Roger, 357
Weinberger, Hans F., 428
Weyl's inequalities, 310
Weyl, Herman, 310, 323
Wilkinson, John N., 460
Woerdeman, Hugo J., 422, 436
Wolkowicz, Henry, 422
Wronskian, 232
- Young's inequality, 95
Young, Gale, 170
- zero matrix, 3
Zhang, F., 391
Zhou, Kemin, 428, 447

SELECTED PUBLISHED TITLES IN THIS SERIES

- 232 **Harry Dym**, *Linear Algebra in Action*, Third Edition, 2023
- 231 **Luís Barreira and Yakov Pesin**, *Introduction to Smooth Ergodic Theory*, Second Edition, 2023
- 229 **Giovanni Leoni**, *A First Course in Fractional Sobolev Spaces*, 2023
- 228 **Henk Bruin**, *Topological and Ergodic Theory of Symbolic Dynamics*, 2022
- 227 **William M. Goldman**, *Geometric Structures on Manifolds*, 2022
- 226 **Milivoje Lukić**, *A First Course in Spectral Theory*, 2022
- 225 **Jacob Bedrossian and Vlad Vicol**, *The Mathematical Analysis of the Incompressible Euler and Navier-Stokes Equations*, 2022
- 224 **Ben Krause**, *Discrete Analogues in Harmonic Analysis*, 2022
- 223 **Volodymyr Nekrashevych**, *Groups and Topological Dynamics*, 2022
- 222 **Michael Artin**, *Algebraic Geometry*, 2022
- 221 **David Damanik and Jake Fillman**, *One-Dimensional Ergodic Schrödinger Operators*, 2022
- 220 **Isaac Goldbring**, *Ultrafilters Throughout Mathematics*, 2022
- 219 **Michael Joswig**, *Essentials of Tropical Combinatorics*, 2021
- 218 **Riccardo Benedetti**, *Lectures on Differential Topology*, 2021
- 217 **Marius Crainic, Rui Loja Fernandes, and Ioan Mărcuț**, *Lectures on Poisson Geometry*, 2021
- 216 **Brian Osserman**, *A Concise Introduction to Algebraic Varieties*, 2021
- 215 **Tai-Ping Liu**, *Shock Waves*, 2021
- 214 **Ioannis Karatzas and Constantinos Kardaras**, *Portfolio Theory and Arbitrage*, 2021
- 213 **Hung Vinh Tran**, *Hamilton–Jacobi Equations*, 2021
- 212 **Marcelo Viana and José M. Espinar**, *Differential Equations*, 2021
- 211 **Mateusz Michałek and Bernd Sturmfels**, *Invitation to Nonlinear Algebra*, 2021
- 210 **Bruce E. Sagan**, *Combinatorics: The Art of Counting*, 2020
- 209 **Jessica S. Purcell**, *Hyperbolic Knot Theory*, 2020
- 208 **Vicente Muñoz, Ángel González-Prieto, and Juan Ángel Rojo**, *Geometry and Topology of Manifolds*, 2020
- 207 **Dmitry N. Kozlov**, *Organized Collapse: An Introduction to Discrete Morse Theory*, 2020
- 206 **Ben Andrews, Bennett Chow, Christine Guenther, and Mat Langford**, *Extrinsic Geometric Flows*, 2020
- 205 **Mikhail Shubin**, *Invitation to Partial Differential Equations*, 2020
- 204 **Sarah J. Witherspoon**, *Hochschild Cohomology for Algebras*, 2019
- 203 **Dimitris Koukoulopoulos**, *The Distribution of Prime Numbers*, 2019
- 202 **Michael E. Taylor**, *Introduction to Complex Analysis*, 2019
- 201 **Dan A. Lee**, *Geometric Relativity*, 2019
- 200 **Semyon Dyatlov and Maciej Zworski**, *Mathematical Theory of Scattering Resonances*, 2019
- 199 **Weinan E, Tiejun Li, and Eric Vanden-Eijnden**, *Applied Stochastic Analysis*, 2019
- 198 **Robert L. Benedetto**, *Dynamics in One Non-Archimedean Variable*, 2019
- 197 **Walter Craig**, *A Course on Partial Differential Equations*, 2018
- 196 **Martin Stynes and David Stynes**, *Convection-Diffusion Problems*, 2018
- 195 **Matthias Beck and Raman Sanyal**, *Combinatorial Reciprocity Theorems*, 2018
- 194 **Seth Sullivant**, *Algebraic Statistics*, 2018
- 193 **Martin Lorenz**, *A Tour of Representation Theory*, 2018

For a complete list of titles in this series, visit the
AMS Bookstore at www.ams.org/bookstore/gsmseries/.