Preface to the third edition

This new edition of *Linear Algebra in Action* is significantly different from the previous edition in both content and style: It includes a number of topics that did not appear in the earlier edition and excludes some that did. In the earlier edition I entered into the proofs of every fact that was used. In this edition I have relegated the proofs of a number of theorems to outside references and have focused instead on their applications, which to my mind has more impact than a proof, especially on a first pass. Moreover, most of the material that is adapted from the previous edition has been rewritten and reorganized.

I have organized this book into short chapters, most of which are a dozen pages or less, because I think this is more amenable to classroom use. I have tried to write this book in the style that most of the mathematicians that I know work in, rather than in the way that they write. In particular, I believe that the discussion of a well-chosen example is often much more helpful than a formal proof, which in many cases is an example hidden by elaborate bookkeeping. A good student will be able to pass to the general setting from the example, and a weaker student will at least have something concrete to focus on.

The book is intended primarily for students who have had at least a little exposure to linear algebra. Nevertheless, the first twelve chapters or so are basically a quick review of the material that is typically offered in a first course, plus a little. The content of this introductory material is dealt with in greater detail in the second edition of this book, which is a useful supplement to this third edition, but the two can be used independently.

A reader who is familiar with the main contents of the first sixteen chapters and Chapter 21 should be able to read any of the other chapters without difficulty, as they are for the most part independent of each other.

The entries **keep in mind**, **warning**, and **notation** appear in the index. The first is to call attention to compilations that I feel are helpful, the second is to call attention to conventions that have been introduced, and the third is to point out the introduction of new notation, most of which is fairly standard, except that a distinction is made between the matrices A^H (the conjugate transpose of A) and A^* (the adjoint of A, which depends upon the inner product).

I extend my thanks to the readers who contributed corrections to the first two editions over the past several years and to Shmuel Aviya for reading and commenting on a number of chapters as they were being prepared for this third edition. I owe a special note of thanks to Dr. Andrei Iacob who carefully copyedited a close to final version of the third edition. It is also a pleasure to thank the staff of the AMS for their friendly help, with extra special thanks to my copyeditor, Arlene O'Sean, for accommodating the author and for her care and devotion to getting things right.

I plan to use the AMS website `www.ams.org/bookpages/gsm-232` to supply some supplementary material as well as for sins of omission and commission (and just plain afterthoughts).

I did not add *words of wisdom* at the beginning of each chapter as in the earlier editions. However, I cannot resist repeating two of them:

The first is based on some four score and five years of observing the human scene:

Those who think they know all the answers don't know all the questions.

A Chinese proverb puts it well: *Trust only those who doubt.*

The second is one that I am especially fond of (both the saying and its originator, who was a very special person), though I have never been able to live up to it:

Let's throw everything away; then there will be room for what's left.

–Irene Dym

TAM ACH TEREM NISHLAM

February 14, 2023

Rehovot, Israel

Preface to the second edition

I have an opinion. But I do not agree with it.

Joshua Sobol [**83**]

Most of the chapters in the first edition have been revised, some extensively. The revisions include changes in a number of proofs, to either simplify the argument and/or make the logic clearer, and, on occasion, to sharpen the result.

New short introductory sections on linear programming, extreme points for polyhedra and a Nevanlinna-Pick interpolation problem have been added, as have some very short introductory sections on the mathematics behind Google, Drazin inverses, band inverses, and applications of svd together with a number of new exercises.

I would like to thank the many readers who e-mailed me helpful lists of typographical errors. I owe a special word of thanks to David Kimsey and Motke Porat, whose lists hit double figures. I believe I have fixed all the reported errors and then some.

A couple of oversights in the first edition that came to light (principally the fact that the word Hankel should be removed from the statement and proof of Corollary 21.2; an incomplete definition of a support hyperplane; and a certain fuzziness in the discussion of operator norms and multiplicative norms) have also been fixed.

It is a pleasure to thank the staff of the AMS for being so friendly and helpful; a special note of thanks to my copy/production editor Mike Saitas for his sharp eye and cheerful willingness to accommodate the author and to Mary Medeiros for preparing the indices and her expertise in LaTeX.

The AMS website www.ams.org/bookpages/gsm-78 will be used for sins of omission and commission (and just plain afterthoughts) for the second edition as well as the first.

TAM, ACH TEREM NISHLAM, ...

July 19, 2013

Rehovot, Israel

Preface to the first edition

A foolish consistency is the hobgoblin of little minds,...

Ralph Waldo Emerson, *Self Reliance*

This book is based largely on courses that I have taught at the Feinberg Graduate School of the Weizmann Institute of Science over the past 35 years to graduate students with widely varying levels of mathematical sophistication and interests. The objective of a number of these courses was to present a user-friendly introduction to linear algebra and its many applications. Over the years I wrote and rewrote (and then, more often than not, rewrote some more) assorted sets of notes and learned many interesting things en route. This book is the current end product of that process. The emphasis is on developing a comfortable familiarity with the material. Many lemmas and theorems are made plausible by discussing an example that is chosen to make the underlying ideas transparent in lieu of a formal proof; i.e., I have tried to present the material in the way that most of the mathematicians that I know work rather than in the way they write. The coverage is not intended to be exhaustive (or exhausting), but rather to indicate the rich terrain that is part of the domain of linear algebra and to present a decent sample of some of the tools of the trade of a working analyst that I have absorbed and have found useful and interesting in more than 40 years in the business. To put it another way, I wish someone had taught me this material when I was a graduate student. In those days, in the arrogance of youth, I thought that linear algebra was for boys and girls and that real

men and women worked in functional analysis. However, this is but one of many opinions that did not stand the test of time.

In my opinion, the material in this book can (and has been) used on many levels. A core course in classical linear algebra topics can be based on the first six chapters, plus selected topics from Chapters 7–9 and 13. The latter treats difference equations, differential equations, and systems thereof. Chapters 14–16 cover applications to vector calculus, including a proof of the implicit function based on the contractive fixed point theorem, and extremal problems with constraints. Subsequent chapters deal with matrix-valued holomorphic functions, matrix equations, realization theory, eigenvalue location problems, zero location problems, convexity, and matrices with nonnegative entries. I have taken the liberty of straying into areas that I consider significant, even though they are not usually viewed as part of the package associated with linear algebra. Thus, for example, I have added short sections on complex function theory, Fourier analysis, Lyapunov functions for dynamical systems, boundary value problems and more. A number of the applications are taken from control theory.

I have adapted material from many sources. But the one which was most significant for at least the starting point of a number of topics covered in this work is the wonderful book [**56**] by Lancaster and Tismenetsky.

A number of students read and commented on substantial sections of assorted drafts: Boris Ettinger, Ariel Ginis, Royi Lachmi, Mark Kozdoba, Evgeny Muzikantov, Simcha Rimler, Jonathan Ronen, Idith Segev, and Amit Weinberg. I thank them all, and extend my appreciation to two senior readers: Aad Dijksma and Andrei Iacob for their helpful insightful remarks. A special note of thanks goes to Deborah Smith, my copy editor at the AMS, for her sharp eye and expertise in the world of commas and semicolons.

On the production side, I thank Jason Friedman for typing an early version, and our secretaries Diana Mandelik, Ruby Musrie, Linda Alman, Terry Debesh, all of whom typed selections and to Diana again for preparing all the figures and clarifying numerous mysterious intricacies of LaTeX. I also thank Barbara Beeton of the AMS for helpful advice on AMS LaTeX.

One of the difficulties in preparing a manuscript for a book is knowing when to let go. It is always possible to write it better.[1] Fortunately the

[1]Israel Gohberg tells of a conversation with Lev Sakhnovich that took place in Odessa many years ago: Lev: Israel, how is your book with Mark Gregorovic (Krein) progressing? Israel: It's about 85% done. Lev: That's great! Why so sad? Israel: If you would have asked me yesterday, I would have said 95%.

AMS maintains a web page: http://www.ams.org/bookpages/gsm-78, for sins of omission and commission (or just plain afterthoughts).

TAM, ACH TEREM NISHLAM,...

October 18, 2006

Rehovot, Israel