
Contents

Preface to the Second Edition	xiii
Preface to the First Edition	xv
Intrigue	1
Part 1. Holonomic Approximation	
Chapter 1. Jets and Holonomy	9
§1.1. Maps and sections	9
§1.2. Coordinate definition of jets	10
§1.3. Invariant definition of jets	11
§1.4. The space $X^{(1)}$	12
§1.5. Holonomic sections of the jet space $X^{(r)}$	13
§1.6. Geometric representation of sections of $X^{(r)}$	14
§1.7. Holonomic splitting	14
Chapter 2. Thom Transversality Theorem	17
§2.1. Generic properties and transversality	17
§2.2. Stratified sets and polyhedra	18
§2.3. Thom Transversality Theorem	19
Chapter 3. Holonomic Approximation	25
§3.1. Main theorem	25
§3.2. Holonomic approximation over a cube	28
§3.3. Holonomic extension	28

§3.4.	Gluing Lemma	30
§3.5.	Proof of Theorem 3.2.1	33
§3.6.	Proof of the Gluing Lemma	35
§3.7.	Parametric holonomic approximation	40
§3.8.	Foliated holonomic approximation	41
§3.9.	Refinement of the Holonomic Approximation Theorem	43
Chapter 4.	Applications	45
§4.1.	Functions without critical points	45
§4.2.	Smale's sphere eversion	46
§4.3.	Approximate integration of tangential homotopies	48
§4.4.	Open manifolds	50
§4.5.	Directed embeddings of open manifolds	53
§4.6.	Directed embeddings of closed manifolds	55
§4.7.	Approximation of differential forms by closed forms	56
Chapter 5.	Multivalued Holonomic Approximation	59
§5.1.	Multifolds	59
§5.2.	Holonomic approximation by multivalued sections	63
Part 2. Differential Relations and Gromov's h-principle		
Chapter 6.	Differential Relations	69
§6.1.	What is a differential relation?	69
§6.2.	Open and closed differential relations	71
§6.3.	Formal and genuine solutions to a differential relation	72
§6.4.	Extension problem	72
§6.5.	Approximate solutions to systems of differential equations	73
Chapter 7.	Homotopy Principle	75
§7.1.	Philosophy of the h -principle	75
§7.2.	Different flavors of the h -principle	78
Chapter 8.	Open Diff V -invariant Differential Relations	81
§8.1.	Natural fibrations	81
§8.2.	Diff V -invariant differential relations	85
§8.3.	Local and global h -principle for open Diff V -invariant relations	85
Chapter 9.	Applications to Closed Manifolds	89
§9.1.	Microextension trick	89

§9.2. Smale–Hirsch h -principle	89
§9.3. Sections transverse to a distribution	91
§9.4. Multivalued h -principle	93
Chapter 10. Foliations	95
§10.1. Definition and examples	95
§10.2. Haefliger structures and h -principle for foliations	98
§10.3. Foliated h -principle	102
 Part 3. Singularities and Wrinkling	
Chapter 11. Singularities of Smooth Maps	109
§11.1. Thom–Boardman singularities	109
§11.2. The Morse Lemma	112
§11.3. Tangency of a submanifold to a foliation	114
§11.4. Fibered character of Σ^1 -singularities	115
§11.5. A_n -singularities of functions	118
§11.6. Morin’s normal forms	119
§11.7. More about the geometry of maps with $\Sigma^{1,\dots,1}$ -singularities	123
Chapter 12. Wrinkles	127
§12.1. Standard wrinkles	127
§12.2. Properties of standard wrinkles	129
Chapter 13. Wrinkled Submersions	135
§13.1. Definitions	135
§13.2. Main results	138
§13.3. Wrinkled immersion associated with a multifold	142
§13.4. Chopping wrinkles	144
§13.5. Corrugation	148
Chapter 14. Folded Solutions to Differential Relations	155
§14.1. Surgery of singularities	155
§14.2. Folded solutions to differential relations	164
Chapter 15. The h -principle for Sharp Wrinkled Embeddings	169
§15.1. Sharp wrinkled embeddings	169
§15.2. Integration of tangential rotations	176
§15.3. Preliminary steps in the proof	177
§15.4. Reduction to the main lemma	179

§15.5.	Proof of the main lemma	181
§15.6.	The h -principle for folded embeddings	187
Chapter 16.	Igusa Functions	193
§16.1.	Leafwise Igusa functions and their formal analogues	193
§16.2.	Formal extension of framed ζ -FLIFs	198
§16.3.	Constructing a locally holonomic ζ -FLIF	199
§16.4.	Proof of the main theorems	208
§16.5.	Applications to pseudo-isotopy theory	213
Part 4. The Homotopy Principle in Symplectic Geometry		
Chapter 17.	Symplectic and Contact Basics	219
§17.1.	Linear symplectic and complex geometries	219
§17.2.	Symplectic and complex manifolds	224
§17.3.	Symplectic stability	229
§17.4.	Contact manifolds	232
§17.5.	Contact stability	237
§17.6.	Lagrangian and Legendrian submanifolds	240
§17.7.	Hamiltonian and contact vector fields	241
§17.8.	Characteristic foliations	243
Chapter 18.	Symplectic and Contact Structures on Open Manifolds	245
§18.1.	Classification problem for symplectic and contact structures	245
§18.2.	Symplectic structures on open manifolds	246
§18.3.	Contact structures on open manifolds	248
§18.4.	Two-forms of maximal rank on odd-dimensional manifolds	249
Chapter 19.	Symplectic and Contact Structures on Closed Manifolds	251
§19.1.	Symplectic structures on closed manifolds	251
§19.2.	Contact structures on closed manifolds	253
§19.3.	Folded symplectic and contact structures	255
Chapter 20.	Embeddings into Symplectic and Contact Manifolds	259
§20.1.	Isosymplectic embeddings	259
§20.2.	Equidimensional isosymplectic immersions	268
§20.3.	Isocontact embeddings	271
§20.4.	Subcritical isotropic embeddings	276

Chapter 21. Microflexibility and Holonomic \mathcal{R} -approximation	279
§21.1. Local integrability	279
§21.2. Homotopy extension property for formal solutions	281
§21.3. Microflexibility	281
§21.4. Theorem on holonomic \mathcal{R} -approximation	283
§21.5. Local h -principle for microflexible Diff V -invariant relations	283
Chapter 22. First Applications of Microflexibility	285
§22.1. Subcritical isotropic immersions	285
§22.2. Maps transverse to a contact structure	286
Chapter 23. Microflexible \mathfrak{A} -invariant Differential Relations	289
§23.1. \mathfrak{A} -invariant differential relations	289
§23.2. Local h -principle for microflexible \mathfrak{A} -invariant relations	290
Chapter 24. Further Applications to Symplectic Geometry	293
§24.1. Legendrian and isocontact immersions	293
§24.2. Generalized isocontact immersions	294
§24.3. Lagrangian immersions	296
§24.4. Isosymplectic immersions	297
§24.5. Generalized isosymplectic immersions	299
Part 5. Convex Integration	
Chapter 25. One-Dimensional Convex Integration	303
§25.1. Example	303
§25.2. Convex hulls and ampleness	304
§25.3. Main lemma	305
§25.4. Proof of the main lemma	306
§25.5. Parametric version of the main lemma	311
§25.6. Proof of the parametric version of the main lemma	312
Chapter 26. Homotopy Principle for Ample Differential Relations	317
§26.1. Ampleness in coordinate directions	317
§26.2. Iterated convex integration	318
§26.3. Principal subspaces and ample differential relations in $X^{(1)}$	320
§26.4. Convex integration of ample differential relations	321
Chapter 27. Directed Immersions and Embeddings	323
§27.1. Criterion of ampleness for directed immersions	323

§27.2.	Directed immersions into almost symplectic manifolds	324
§27.3.	Directed immersions into almost complex manifolds	325
§27.4.	Directed embeddings	326
Chapter 28.	First Order Linear Differential Operators	331
§28.1.	Formal inverse of a linear differential operator	331
§28.2.	Homotopy principle for \mathcal{D} -sections	332
§28.3.	Nonvanishing \mathcal{D} -sections	333
§28.4.	Systems of linearly independent \mathcal{D} -sections	334
§28.5.	Two-forms of maximal rank on odd-dimensional manifolds	336
§28.6.	One-forms of maximal rank on even-dimensional manifolds	338
Chapter 29.	Nash–Kuiper Theorem	341
§29.1.	Isometric immersions and short immersions	341
§29.2.	Nash–Kuiper theorem	342
§29.3.	Decomposition of a metric into a sum of primitive metrics	343
§29.4.	Approximation theorem	344
§29.5.	One-dimensional Approximation Theorem	345
§29.6.	Adding a primitive metric	346
§29.7.	End of the proof of the Approximation Theorem	348
§29.8.	Proof of the Nash–Kuiper theorem	349
Bibliography		351
Index		359