Preface

This book aims to cover basic results in the Monge–Ampère equation, the linearized Monge–Ampère equation, and their applications. It expands upon materials of special topic courses (M742) that I taught at Indiana University, Bloomington, in the spring semesters of 2016 and 2020.

The Monge–Ampère equation has been studied for a long time and is still a center of intensive interest nowadays because of its significant role in many important problems in analysis, geometry, physics, partial differential equations, and applications. Its linearization, that is, the linearized Monge–Ampère equation, has attracted considerable attention in recent years. It arises from several fundamental problems in different subjects such as affine geometry, complex geometry, meteorology, and the calculus of variations.

For the Monge–Ampère equation, I present a wide range of fundamental results and phenomena in the solvability and regularity theory for Aleksandrov solutions with more or less optimal assumptions on the data. The regularity theory is considered both in the interior and at the boundary. Thus, this book complements the interior regularity theory treated in standard textbooks on the modern theory of the Monge–Ampère equation including the classical book *The Monge–Ampère Equation* by Gutiérrez and the recent, elegant book *The Monge–Ampère Equation and Its Application* by Figalli. In addition, it also introduces the spectral theory of the Monge–Ampère operator. Moreover, due to their role in diverse applications, considerable attention is paid to the Monge–Ampère equations in their very natural setting, that is, general convex domains that are not strictly convex.

For the linearized Monge–Ampère equation, my goal here is to give a more or less complete account of the local and global regularity theories in Hölder spaces for solutions. It should be noted that the above-mentioned book by Gutiérrez was also the first to cover the interior Harnack inequality for the linearized Monge–Ampère equation—the fundamental work of Caffarelli and Gutiérrez—which initiated many developments and interesting applications. Around the same time, the book *Dynamical and Geometric Aspects of Hamilton–Jacobi and Linearized Monge–Ampère Equations* by Le, Mitake, and Tran gave an expository account of the Caffarelli–Gutiérrez interior Harnack inequality and a brief introduction to boundary estimates. This book significantly expands the scopes and methods of these two books in its treatment of the linearized Monge–Ampère equation. Furthermore, it also discusses in depth their applications in various areas of mathematics.

As with most literature on the subject, my expositions here on Monge–Ampère equations are primarily based on measure-theoretic, convex geometric, or maximum-principle-based methods. However, alternative methods are also given. For instance, I include discussion of variational methods, and, in addition to the nondivergence form nature of Monge–Ampère equations, I also discuss them from divergence form perspectives.

Except for a few classical results that will be recalled in the second chapter, the rest of this book is self-contained. I have tried to make it a textbook rather than a research monograph. An effort has been made to give motivations and examples of many concepts introduced in the book. Students having a basic background in linear algebra, real analysis, and partial differential equations can follow the book without much difficulties. To facilitate reading for beginners to the subject, complete details of the proofs of all theoretical results are presented while many subtle constructions are explained. These contribute to the length of the book. I hope that researchers in analysis, geometry, partial differential equations, mathematical physics, and the calculus of variations will find this book a useful reference. There are about a hundred and fifty problems for the interested reader. More details on the contents of the book will be presented in the first chapter.

I am extremely grateful to several anonymous reviewers for their critical comments and suggestions. They tremendously helped improve the presentation of the initial manuscript. I would like to thank the National Science Foundation for support via grants DMS-1764248 and DMS-2054686 during the writing of this book. Finally, I would like to thank the AMS, especially Sergei Gelfand, Arlene O'Sean, and Christine Thivierge, for bringing this book into publication.

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Fall 2023, Bloomington