Index

absolute value, 50
AGE cone, 247  
    conditional, 253  
Archimedean, 156
arithmetic-geometric mean inequality,  
    102  
    weighted, 242
Artin–Schreier theorem, 109
balanced vector, 252
barrier function, 84  
    self-concordant, 93
barrier method, 84
Bernstein polynomial, 132, 135
biduality, 92
Bohnenblust’s theorem, 269
Boolean combination, 24, 40, 45, 47
boundary  
    relative, 265
Budan–Fourier theorem, 5, 17, 19

CAD, see cylindrical algebraic decomposition
central path, 76
certificate, 98  
    of nonnegativity, 101
Choleski decomposition, 145, 267
circuit  
    normalized, 256
    simplicial, 254, 256
    sublinear, see sublinear circuit
circuit number, 255
closure

topological, 266
common interlacer, see interlacer
companion matrix, 11
complexity, 271
concave, 266  
    strictly, 266
cone  
    convex, 80, 106
    dual, 80
    exponential, see exponential cone  
    homogeneous, 84
    ice cream, 81
    pointed, 80
    proper, 80
    relative entropy, see relative entropy  
    cone
    symmetric, 84
conic optimization, see optimization, conic
constrained polynomial optimization,  
    see polynomial optimization,  
    constrained
convergence  
    finite, 163
convex, 264, 266  
    rigidly, see rigid convexity
    strictly, 266
copositive, 274
cylindrical algebraic decomposition, 53,  
    66, 68, 71
    adapted, 67, 71
delineable, 61

289
Descartes's Rule of Signs, 4, 7, 16, 17, 19, 29, 242

determinant polynomial, 70
determinantal representation, 226
determinate, 271
Dirac measure, 150
discriminant, 18, 107, 135
duality, 77, 151
   strong, 252
duality gap, 78
duality theorem
   strong, 74, 78, 82, 150
   weak, 74, 78, 82

ellipsoid, 29
ellipsoid algorithm, 75
elliptope, 27, 30, 31
entropy function, 240
even, 246
exponential cone, 237, 260, 261
exponential polynomial, 241
exponential sum, 241
Extension Lemma, 113
extremal, 106

face, 183
   exposed, 183
facet, 23
Farkas's lemma, 23, 75, 112, 121, 127
   for spectrahedra, 183
feasible, 73, 77
   strictly, 82
feasible region, 73
Féjer's theorem, 78, 268
Fell–Dedieu theorem, 207, 234
field
   algebraically closed, 263
   formally real, 35
   ordered, 34, 52
   real, 35, 52
   real closed, 36
field extension, 35
first-order formula, 47
flat extension, 170
formula
   first order, see first-order formula
   quantifier-free, 48
Fourier sequence, 5
Fourier–Motzkin elimination, 22
Frobenius norm, 268
function
   nondegenerate self-concordant, 89
   self-concordant, 89, 93
Fundamental Theorem of Algebra, 38
Gauß–Lucas theorem, 203
generating polynomial, 208
Goursat transform, 99, 131, 135
$H$-polyhedron, 21
$H$-presentation, see half-space
   presentation, 196
Hadamard product, 76, 88
Handelman's theorem, 121, 127, 134, 135, 138
Hankel matrix, 14, 15, 18, 153
   generalized, 14
Hausdorff moment problem, 140, 269, 270
Haviland's theorem, 154, 173
Helton–Nie conjecture, 196, 198
Helton–Vinnikov theorem, 228
Hermite form, 11, 14, 19
Hermite–Biehler theorem, 214
Hermite–Kakeya–Obreschkoff theorem, 206
   multivariate, 219
Hilbert's 17th Problem, 97, 108, 114, 118, 147
Hilbert's Classification Theorem, 104, 135
Hilbert's Nullstellensatz, 109, 115, 116
   strong, 264
   weak, 110, 263
homogeneous
   logarithmically, 88
Hurwitz matrix, 216
Hurwitz stable, 213
Hurwitz's theorem, 219
hyperbolic, see polynomial, hyperbolic
   hyperbolic programming, 231, 235
   hyperbolicity cone, 223
ideal, 263
independent set, see stable set
infeasibility certificate, 111, 115, 187
integration, 148
interior
   algebraic, 178
   relative, 265, see relative interior
interior point method, 76, 79, 84
interlaced, see polynomial, interlaced, 214
interlacer
common, 207
intermediate value property, 38
Jacobi–Prestel theorem, 128
Khatri–Rao product, 193
Krivine–Stengle Positivstellensatz, see Positivstellensatz
Kronecker product, 269
Kullback–Leibler divergence, 241
Lasserre’s hierarchy, 156, 160
Lax conjecture, 228, 235
linear matrix inequality, 27
linear optimization
relaxation, 138
linear programming, see optimization, linear
LMI, see linear matrix inequality
localization form, 159
truncated, 159
log-concave, 208
Lorentz cone, see second-order cone, 224
Lorentz degree, 132
Lorentzian polynomial, 235
LP, see optimization, linear
matching, 210
matching polynomial, 210
matrix
copositive, see copositive
positive definite, see positive definite
positive semidefinite, see positive semidefinite
square root, 267
symmetric, 267
matrix pencil, 27, 176
reduced, 176
matrix polynomial
linear, 27, 176
measure, 139, 147
atomic, 168
minor, 267
leading principal, 267
principal, 267
moment, 269
moment cone, 152
moment form, 148
moment matrix, 148
truncated, 148
moment problem, 151
Hausdorff, see Hausdorff moment problem
moment relaxation, 148, 160
moment sequence, 140
monic, 11, 175
monoid, 115
Motzkin polynomial, 102, 104, 133, 144, 147, 171
Motzkin–Straus theorem, 272
Newton identities, 13, 14
Newton polytope, 102, 112, 171
Newton sum, 11, 13, 16
generalized, 18
Newton–Girard formulas, see Newton identities
nonnegative, 22
nonnegative polynomial, 101
norm
Frobenius, 77
NP, 271
NP-complete, 271
strongly, 272
NP-hard, 271
Nullstellensatz
Hilbert’s, see Hilbert’s Nullstellensatz
real, see Real Nullstellensatz
open quadrant problem, 51, 52
optimality, 74
detection, 163
optimization, ix
conic, 80, 93
linear, 73, 93
polynomial, see polynomial optimization
semidefinite, 76
order, 34, 109
partial, 33
total, 33
order extension, 35
partition problem, 272
perspective function, 238
Petersen graph, 211
Pólya’s theorem, 119, 135
Pólya–Szegő theorem, 100, 131, 135
polyhedron, 21, 31
polynomial, 263
characteristic, 11, 28, 31, 107
elementary symmetric, 18
hyperbolic, 222, 235
inequalities, 115
interlaced, 203
nonnegative, 98, 99, 151, 274
positive, 97–99
real, 3
real stable, 202
real zero, see real zero polynomial, 228
real-rooted, 202, 208
stable, 202, 234
symmetric, 10
univariate, 3
polynomial optimization, 137, 272
polytope, 21, 31, 139
neighborly, 24
positive definite, 267
positive hull, 249
positive semidefinite, 16, 27, 267
Positivstellensatz, 110, 112, 115, 118, 135, 137
power sum, 11
preorder, 34, 100, 109, 111, 115
proper, 111
principal subresultant coefficient, 58
program
conic, 80
linear, 73
semidefinite, 77
projection
of a polyhedron, 21
of a semialgebraic set, 26
projection phase, 68
Projection Theorem, 26, 46
proper position, 204
Puiseux series
real, 36
Putinar's theorem, 124, 127, 130, 131, 135, 156
dual, 162
quadratic form, 14
rank, 14
signature, 14
quadratic module, 111, 135, 187
Archimedean, 123, 191
maximal, 113
proper, 111
truncated, 156
quantifier elimination, 26, 48
radical
real, 167
radical ideal, 263
real, see real radical ideal
real algebraic geometry, ix
real algebraic numbers, 36
Real Nullstellensatz, 133
strong, 118
weak, 115
real radical ideal, 118
real variety, see variety, real
real zero polynomial, 177
real-rooted, see polynomial, real-rooted
relative entropy cone, 240
relative entropy function, 240
relative interior, 83, 266
resultant, 56, 70, 71
Riesz functional, 140
rigid convexity, 177, 178, 181, 197
ring of polynomials, 263
Robinson polynomial, 171
Rolle’s theorem, 39
root, 3
root counting, 3
root function, 64
root isolation, 10
Routh–Hurwitz problem, 212
Rule of Signs
Descartes’s, see Descartes’s Rule of Signs
RZ polynomial, see real zero polynomial
SAGE cone, 247
conditional, 253, 262
Schmüdgen’s theorem, 129, 131, 135
Schur complement, 29, 268
SDFP, see semidefinite feasibility problem
second-order cone, 29, 93
self-dual, 268
semialgebraic, 26, 195
semialgebraic set, 24, 26, 115
basic, 100, 137, 155
basic closed, 24
semidefinite feasibility problem, 79, 93, 184, 197
semidefinite program, see program, semidefinite
semidefinite programming, 31, see optimization, semidefinite
semidefinitely representable set, 194
separable, 265
strictly, 265
Separation Theorem, 83, 264
Index

sign invariant, 67
sign matrix, 41
signomial, 241
simplex, 30
simplex algorithm, 75
slack variable, 76
spectrahedral shadow, 194, 196, 198
spectrahedron, 27, 31, 175, 176, 181
  bounded, 175, 191
  containment, 175, 191, 198
  empty, 175, 188
stable, 217
  Hurwitz, see Hurwitz stable
  real, 217
stable polynomial, see polynomial, stable
stable set, 272
Stodola’s criterion, 213
Sturm sequence, 7, 17, 19
sublinear circuit, 258
subresultant, 58, 71
subresultant coefficient, see principal subresultant coefficient
subresultant matrix, 58, 70
subresultant polynomial, 70
sum of squares, 98, 103, 104, 110, 143
  relaxation, 143, 147, 157
support function, 251
supporting hyperplane, 264
Sylvester matrix, 56
Sylvester’s law of inertia, 15
Tarski’s transfer principle, 49, 109, 114
Tarski–Seidenberg principle, 33, 40, 47, 52
ternary quartic, 104
transfer principle, see Tarski’s transfer principle
TV screen, 180, 194

unconstrained optimization, 143
unimodal, 208
unique factorization domain, 57
upper bound theorem, 23, 31

\(V\)-polytope, 21
\(V\)-presentation, see vertex presentation
vanishing ideal, 263
variety, 263
  real, 24, 115
vertex, 23
vertex presentation, 21
## Selected Published Titles in This Series

<table>
<thead>
<tr>
<th>Title</th>
<th>Author(s)</th>
<th>Publication Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Algebraic Geometry and Optimization</td>
<td>Thorsten Theobald</td>
<td>2024</td>
</tr>
<tr>
<td>Topics in Spectral Geometry</td>
<td>Michael Levitin, Dan Mangoubi, and Isiof Polterovich</td>
<td>2023</td>
</tr>
<tr>
<td>Ricci Solitons in Low Dimensions</td>
<td>Bennett Chow</td>
<td>2023</td>
</tr>
<tr>
<td>Homological Methods in Commutative Algebra</td>
<td>Andrea Ferretti</td>
<td>2023</td>
</tr>
<tr>
<td>Commutative Algebra</td>
<td>Andrea Ferretti</td>
<td>2023</td>
</tr>
<tr>
<td>Linear Algebra in Action, Third Edition</td>
<td>Harry Dym</td>
<td>2023</td>
</tr>
<tr>
<td>Introduction to Smooth Ergodic Theory, Second Edition</td>
<td>Luís Barreira and Yakov Pesin</td>
<td>2023</td>
</tr>
<tr>
<td>Inverse Problems for Fractional Partial Differential Equations</td>
<td>Barbara Kaltenbacher and William Rundell</td>
<td>2023</td>
</tr>
<tr>
<td>A First Course in Fractional Sobolev Spaces</td>
<td>Giovanni Leoni</td>
<td>2023</td>
</tr>
<tr>
<td>Topological and Ergodic Theory of Symbolic Dynamics</td>
<td>Henk Bruin</td>
<td>2022</td>
</tr>
<tr>
<td>Geometric Structures on Manifolds</td>
<td>William M. Goldman</td>
<td>2022</td>
</tr>
<tr>
<td>A First Course in Spectral Theory</td>
<td>Milivoje Lukić</td>
<td>2022</td>
</tr>
<tr>
<td>The Mathematical Analysis of the Incompressible Euler and Navier-Stokes Equations</td>
<td>Jacob Bedrossian and Vlad Vicol</td>
<td>2022</td>
</tr>
<tr>
<td>Discrete Analogues in Harmonic Analysis</td>
<td>Ben Krause</td>
<td>2022</td>
</tr>
<tr>
<td>Groups and Topological Dynamics</td>
<td>Volodymyr Nekrashevych</td>
<td>2022</td>
</tr>
<tr>
<td>Algebraic Geometry</td>
<td>Michael Artin</td>
<td>2022</td>
</tr>
<tr>
<td>One-Dimensional Ergodic Schrödinger Operators</td>
<td>David Damanik and Jake Fillman</td>
<td>2022</td>
</tr>
<tr>
<td>Ultrafilters Throughout Mathematics</td>
<td>Isaac Goldbring</td>
<td>2022</td>
</tr>
<tr>
<td>Essentials of Tropical Combinatorics</td>
<td>Michael Joswig</td>
<td>2021</td>
</tr>
<tr>
<td>Lectures on Differential Topology</td>
<td>Riccardo Benedetti</td>
<td>2021</td>
</tr>
<tr>
<td>Lectures on Poisson Geometry</td>
<td>Marius Crainic, Rui Loja Fernandes, and Ioan Mărcuț</td>
<td>2021</td>
</tr>
<tr>
<td>A Concise Introduction to Algebraic Varieties</td>
<td>Brian Osserman</td>
<td>2021</td>
</tr>
<tr>
<td>Shock Waves</td>
<td>Tai-Ping Liu</td>
<td>2021</td>
</tr>
<tr>
<td>Portfolio Theory and Arbitrage</td>
<td>Ioannis Karatzas and Constantinos Kardaras</td>
<td>2021</td>
</tr>
<tr>
<td>Hamilton–Jacobi Equations</td>
<td>Hung Vinh Tran</td>
<td>2021</td>
</tr>
<tr>
<td>Differential Equations</td>
<td>Marcelo Viana and José M. Espinar</td>
<td>2021</td>
</tr>
<tr>
<td>Invitation to Nonlinear Algebra</td>
<td>Mateusz Michałek and Bernd Sturmfels</td>
<td>2021</td>
</tr>
<tr>
<td>Combinatorics: The Art of Counting</td>
<td>Bruce E. Sagan</td>
<td>2020</td>
</tr>
<tr>
<td>Hyperbolic Knot Theory</td>
<td>Jessica S. Purcell</td>
<td>2020</td>
</tr>
<tr>
<td>Geometry and Topology of Manifolds</td>
<td>Vicente Muñoz, Ángel González-Prieto, and Juan Ángel Rojo</td>
<td>2020</td>
</tr>
<tr>
<td>Organized Collapse: An Introduction to Discrete Morse Theory</td>
<td>Dmitry N. Kozlov</td>
<td>2020</td>
</tr>
<tr>
<td>Extrinsic Geometric Flows</td>
<td>Ben Andrews, Bennett Chow, Christine Guenther, and Mat Langford</td>
<td>2020</td>
</tr>
<tr>
<td>Invitation to Partial Differential Equations</td>
<td>Mikhail Shubin</td>
<td>2020</td>
</tr>
<tr>
<td>Hochschild Cohomology for Algebras</td>
<td>Sarah J. Witherspoon</td>
<td>2019</td>
</tr>
<tr>
<td>The Distribution of Prime Numbers</td>
<td>Dimitris Koukoulopoulos</td>
<td>2019</td>
</tr>
</tbody>
</table>

For a complete list of titles in this series, visit the AMS Bookstore at [www.ams.org/bookstore/gsmseries/](http://www.ams.org/bookstore/gsmseries/).