

Index

- $(\mathbb{Z}/N\mathbb{Z})^*$, 181
 $(\mathbb{Z}/N\mathbb{Z})^*$, invertible elements in $\mathbb{Z}/N\mathbb{Z}$, 48
 $E[X]$, expectation of X , 186
 E_C , entanglement cost, 149
 E_D , distillable entanglement, 149
 E_{sq} , squashed entanglement, 152
 $F(\cdot, \cdot)$, fidelity, 89
 G -invariants, 160
 H , Hadamard matrix, 24
 $H(\bar{p}||\bar{q})$, relative entropy, 125
 $H(\bar{p}_y|\bar{p}_x)$, conditional entropy, 90
 $H(\bar{p})$, Shannon entropy, 81
 $H(\rho||\sigma)$, relative von Neumann entropy, 127
 $H(\rho)$, von Neumann entropy, 122
 $H(\rho|\sigma)$, relative von Neumann entropy, 127
 $H(\rho_A | \rho_B)$, conditional von Neumann entropy, 134
 $I(\bar{p}_x : \bar{p}_y)$ mutual information, 92
 O , big O, xii
 $S_\pi V$, irreducible $GL(V)$ -module, 160
 $T_x \Sigma$, tangent space to Σ at x , 32
 W^G , G invariants in W , 160
 X^\dagger , adjoint of operator X , 29
 $Z \cdot X$, induced action, 103
 $[X, Y]$, commutator, 108
 $[\pi]$, irreducible \mathfrak{S}_d -module, 157
 $\Lambda^1(U)$, controlled U , 59
 \bar{X} , complex conjugate, xii
 $\bar{\pi}$, normalized partition, 162
 $\binom{n}{p}$, binomial coefficient, xii
 $\binom{n}{p_1, \dots, p_d}$, multinomial coefficient, xii
 $\chi(\Lambda)$, Holevo capacity, 141
 $\chi(\rho_{XB})$, Holevo χ quantity, 140
 $\ell(\pi)$, length of partition π , 157
 \hbar , Planck's constant, 31
 $\langle v|w \rangle$, inner product, 27
 $\mathbb{Z}/N\mathbb{Z}$, cyclic group, 181
 $O(n)$, orthogonal group, 25
 \mathbf{t} , transpose, xiii
 $\mathcal{D}(\mathcal{H})$, set of density operators on \mathcal{H} , 103
 $\mathcal{D}\mathcal{S}_n$, doubly stochastic matrices, 144
 \mathcal{H} , Hilbert space, 28
 $\mathfrak{u}(\mathcal{H})$, unitary Lie algebra, 32
 ω_N , primitive N -th root of unity, 5
 $\text{var}(X)$, variance of observable, 108
 $\text{Cap}(\Lambda_C)$, capacity of noisy channel, 91
 $\text{Cap}(\Lambda_C, \bar{p})$, capacity of noisy channel, 91
 $\text{End}(V)$, endomorphisms of V , 183
 $\text{Herm}(\mathcal{H})$, Hermitian endomorphisms, 29
 $\text{Hom}_G(V, W)$, 156
 Pr , probability, of an event, 186
 Tof , Toffoli gate, 13
 $\text{conv}(S)$, convex hull of points of S , 144
 $\log(\rho)$, 122
 $\text{mult}(M, W)$, multiplicity of M in W , 160
 spec , spectrum, 146
 $\text{var}(X)$, 186
 \prec , dominance order, 144
 \prec , dominance order for Hermitian operators, 146
 ρ , density operator, 103

- $\sigma(X)$, standard deviation of observable X , 108
- $\sigma(X)$: standard deviation of random variable X , 187
- canonCJ, canonical Choi-Jamiołkowski isomorphism, 114
- CJ, Choi-Jamiołkowski isomorphism, 115
- KRON $_{m,n,k}$, Kronecker polytope, 171
- Kron $_{m,n,k}$, 171
- NOT, 58
- SPEC $_n$, 165
- SPEC $_{m,n,k}$, 172
- YD(π), Young diagram of π , 157
- wt, weight, 164
- $|v|$, length of vector $|v\rangle$, 28
- $|\pi|$, size of partition π , 157
- $\|X\|$, operator norm of X , 57
- $\|X\|_{\text{op}}$, operator norm of X , 57
- $\|\cdot\|$, norm, 56
- $\|\cdot\|_1$, trace norm, 110
- $|\text{epr}\rangle$, epr state, 34
- $c(x)$, content, 160
- $h(x)$, hook length, 160
- $k_{\mu\nu\pi}$, Kronecker coefficient, 167
- o , little o, xii
- p , complexity class, 19
- $v_p(\cdot)$, valuation, 61
- \mathbf{P}/poly , complexity class, 18
- $\mathbf{U}(\mathcal{H})$, unitary group, 29
- $\mathbf{U}(n)$, unitary group, 26
- \otimes , tensor product, 183

- Aaronson, S., 38
- abelian group, 182
- adjoint
 - of operator, 29
- approximation
 - of unitary operator, 58

- big O notation, xii
- binomial coefficient, xii
- bit, 11
- Bloch sphere, 26
- Boolean function, 11

- capacity
 - of noisy channel, 91
- Cartan-Dieudonné theorem, 56
- channel
 - classical, 77
- Chebyshev's inequality, 186
- Chinese Remainder Theorem, 48

- Choi-Jamiołkowski isomorphism, 115
 - canonical, 114
- Church-Turing thesis, 21
- circuit, 11
- circulant matrix, 5
- CNOT, reversible controlled not, 13
- commutator, 108, 184
- completely positive map, 113
- composite system, 33
- concave function, 82
- conditional entropy, 90
- conditional probability, 188
- conditional von Neumann entropy, 134
- content, 160
- continued fraction, 64
- controlled gate, 59
- convolution, 4
- CPTP, 113
 - completely positive trace preserving, 113
- cyclic group, 182

- data processing inequality
 - classical, 92
 - quantum, 140
- density operator, 103
- DFT, 5
- discrete Fourier transform, 5
- distillable entanglement, 149
- dominance order, 144
- doubly stochastic matrices
 - set of, 144
- dual vector space, 183

- entangled density operator, 152
- entangled state, 34
- entanglement, 34
 - distillable, 149
 - of formation, 151
- entanglement catalysis, 149
- entanglement cost, 149
- entanglement monotone, 152
- entropy
 - conditional, 90
 - relative, 125
- EPR, 34
- Erdős, 97
- Euclidean algorithm, 47
- expectation, 186

- fast Fourier transform, 7
- Fermat test for primality, 52
- Fermat Theorem, Little, 50

- FFT, 7
- Fibonacci numbers, 47
- fidelity, 89, 110

- gate, 11
- group, 181
- group homomorphism, 182

- Hadamard, 61
- Hadamard gate, 24
- Hadamard matrix, 24
- Hamiltonian
 - of quantum system, 31
- hay in a haystack, 97
- Hermitian inner product
 - standard, 29
- Hermitian operator, 29
- hidden subgroup problem, 74
- highest weight vector, 170
- Hilbert Schmidt norm, 110
- Hilbert space, 28
- Holevo capacity, 141
- Holweck, F, 37
- hook length, 160
- Hopf fibration, 26

- iid, 187
- induced action
 - of operators on $\text{End}(\mathcal{H})$, 103
- induced group action, 157
- invariants in representation, 160
- irreducible representation, 156
- isomorphism
 - of G -modules, 156
- isotypic component, 160

- Jensen's inequality, 83
- jointly concave function, 135

- Karatsuba, 4
- Karloff, H., 97
- Kepler, 3
- Kolmogorov, 4
- Kraus representation, 116
- Kronecker coefficients, 167
 - semi-group property of, 171
- Kullback-Leibler divergence, 125

- law of large numbers, 187
- law of total probability, 188
- Lie algebra, 32
- Little Fermat Theorem, 50

- majorization, 144

- marginal
 - quantum, 105
- marginal distributions, 185
- Markov's inequality, 186
- measurement
 - POVM, 104
 - projective (von Neumann), 104, 107
- Miller-Rabin test, 63
- mixed state, 105
- Mkrtchyan, S., 162
- module
 - for group, 156
- monotonicity
 - of relative entropy, 138
- multinomial coefficient, xii, 80
- multiplicity of module, 160
- mutual information, 92
 - quantum, 139

- Napier, 3
- Nielson's theorem, 148
- NISQ, x
- no cloning theorem, 151
- Noetherian ring, 178
- norm
 - on vector space, 56

- observable, 107
- operator norm, 57, 136
- order finding algorithm, 69
- orthogonal group, 25

- partial trace, 105
- pbit, 17
- permutation matrix, 15
- Pinsker's inequality, 126
- PIT, 20
- polar decomposition, 111
- polynomial identity testing, 20
- positive endomorphism, 102
- positive operator, 103
- Poussin, 61
- POVM, measurement, 104
- precision of approximation, 58
- prime number theorem, 61
- probability
 - conditional, 188
- probability distribution
 - discrete, 185
 - nonnormalized, 79
- product state, 129
- projection operator, 100

- projective measurement, 104, 107
- pure state, 105
- purification, 106

- quantum channel, 112, 113
 - Kraus representation of, 116
 - Stinespring representation of, 117
- quantum gate set
 - standard, 58
- quantum marginal, 105
- quantum marginal problem, 166
- quantum mutual information, 139
- quantum teleportation, 36
- qubit, 26

- random variable
 - discrete, 185
- random variables
 - identically distributed, 187
 - independent, 187
- recoupling coefficients, 176
- reflection, 43
- register, 13
- relative von Neumann entropy, 127
- reliable transmission, 79
- representation
 - of group, 156
- ring, 182
- ring homomorphism, 183

- SAT, 19
- Schrödinger equation, 30
- Schur's lemma, 156
- Schur-Weyl duality, 160
- separable density operator, 152
- separable state, 34
- Shannon entropy, 81
- singular value, 110
- singular value decomposition, 111
- spectral decomposition theorem, 107
- spectrum estimation theorem, 163
- spectrum of operator, 146
- squashed entanglement, 152
- standard deviation, 108, 187
- standard Hermitian inner-product, 29
- standard purification, 106
- standard quantum gate set, 58
- state space, 30, 104
- state vector, 30
- Stinespring dilation, 117
- Stinespring representation, 117
- Stirling's formula, 62

- stochastic matrix, 17
- subadditivity
 - of Shannon entropy, 86
- submodule, 156
- super-dense coding, 35

- tangent space, 32
- tensor product, 183
- Toffoli gate, 13
- Toffoli matrix, 16
- trace norm, 110
- transpose
 - of linear map, 184
- trivial representation, 156

- unitary group, 29
 - of \mathbb{C}^n , 26
- universal gate set, 12

- Vandermonde determinant, 49
- variance, 186
- variance of Hermitian operator, 108
- Veronese map, 151
- von Neumann entropy, 122
 - conditional, 134
- von Neumann measurement, 104, 107

- weak law of large numbers, 187
- weight, 164
- weight vector, 164
- Wigderson, A., 97
- workspace bit, 14

- Young diagram, 157
- Young tableau
 - default, 158
 - without repetitions, 158