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# Notation Index

- \* (Hodge star operator), 263
- $\infty$  (point at infinity), 56
- $\sharp$  (sharp operator), 229
- $\langle \cdot, \cdot \rangle$  (Hermitian inner product), 76
- $\langle \cdot, \cdot \rangle$  (Kronecker pairing), 166
- $(\cdot, \cdot)$  (global Hodge inner product), 257, 258
- $\|\cdot\|$  (global Hodge norm), 258
- $[\cdot, \cdot]$  (commutator bracket), 281
- $[\cdot]$  (singular homology class), 165
- $[\cdot]$  (cohomology class of a Čech cocycle), 149
- $[[\cdot]]$  (equivalence class in direct limit), 153
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- $A^*$  (cochain complex), 107
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- $B(L)$  (base locus), 95
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- $B_r(p)$  (open ball of radius  $r$ ), 4
- $B(Z, W)$  (holomorphic bisectonal curvature), 253
  
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- $\mathcal{C}^*$  (sheaf of nonvanishing continuous functions), 123
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- $c(L)$  (sheaf-theoretic Chern class), 181
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- $\delta$  (singular coboundary operator), 166
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- $\Delta_{\partial}$  (conjugate Dolbeault Laplacian), 274
- $\Delta_k$  (standard simplex), 164
- $D$  (exterior covariant derivative), 200

- $\mathbb{D}$  (unit disk), 4  
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 $D'F$  (holomorphic Jacobian), 31, 37  
 $DF(p)$  (differential of a smooth map), 30  
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 $D_r(p)$  (disk of radius  $r$ ), 4  
 $d^*$  (formal adjoint of  $d$ ), 262  
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 $\dim_{\mathbb{R}}$  (real dimension), 3  
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 $F_p$  (stalk homomorphism), 128  
 $[f]_p$  (germ of a function), 127  
 $F_{\#}$  (coefficient homomorphism on singular cochains), 166  
 $F_*$  (coefficient homomorphism in singular cohomology), 166  
 $F_*$  (induced sheaf cohomology homomorphism), 153  
 $F^*$  (induced homomorphism on Dolbeault cohomology), 108  
 $\Gamma(E)$  (space of smooth sections of  $E$ ), 25  
 $\Gamma_c(E)$  (space of compactly supported sections), 259  
 $\underline{G}$  (constant sheaf with coefficients in  $G$ ), 123  
 $G/\Gamma$  (coset space), 10  
 $G_k(V)$  (Grassmannian), 6  
 $G_p$  (skyscraper sheaf), 123  
 $g_{\text{CH}}$  (complex hyperbolic metric), 243  
 $g_{\mathbb{E}}$  (standard metric on  $\mathbb{C}^n$ ), 225  
 $g_{\text{FS}}$  (Fubini–Study metric), 235  
 $\text{GL}(n, \mathbb{C})$  (group of invertible complex matrices), 10  
 $\text{GL}(V)$  (group of linear automorphisms of  $V$ ), 10  
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 $\bar{j}$  (barred index), 227  
 $J_M$  (almost complex structure), 36  
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 $\widehat{K}$  (holomorphic hull), 65  
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 $L_{\mathbb{C}}$  (complexification of a linear map), 23  
 $L_{\omega}$  (Lefschetz operator), 281  
 $L_{\{p\}}$  (point bundle), 93  
 $\lim_{\rightarrow} G_{\alpha}$  (direct limit), 128  
 $[M]$  (fundamental homology class), 165  
 $M/\Gamma$  (quotient space by group action), 10  
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 $\text{Pic}^0(M)$  (Picard variety), 182  
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 $r_V^U$  (restriction map in a presheaf), 121  
 $Rc$  (Ricci curvature), 245  
 $Rm$  (Riemann curvature tensor), 239  
 $(\sigma)$  (divisor of a section of a line bundle), 94, 211  
 $\sum'_I$  (sum over increasing multi-indices), 105  
 $S$  (scalar curvature), 245  
 $\widehat{\mathcal{S}}$  (sheaf of rough sections), 186  
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 $\mathcal{S}(K)$  (sections of  $\mathcal{S}$  over a closed subset, 190  
 $\mathcal{S} \otimes_{\mathcal{R}} \mathcal{T}$  (tensor product sheaf), 134  
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 $[s]_p$  (germ of a section), 128  
 $\mathcal{S}(U)$  (sections of  $\mathcal{S}$  over an open subset), 121  
 $\mathcal{S}|_V$  (restriction of a sheaf to an open subset), 123  
 $s|_V$  (restriction of a section), 122  
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 $T'_p M$  (holomorphic tangent space), 37  
 $T''_p M$  (antiholomorphic tangent space), 37  
 $U(n)$  (unitary group), 68  
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 $V''$  ( $(-i)$ -eigenspace of complex structure), 34

$\bar{V}^*$  (space of conjugate-linear functionals),  
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$V_{\mathbb{R}}$  (underlying real vector space), 34

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$\bar{z}^j$  (conjugate of  $z^j$ ), 227

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