
Index

- 1-form, 75

- adjoint representation, 420
- affine graph, 17, 52
- affine hyperplane, 10
- Alexander horned sphere, 47
- alternating map, 328
- alternating tensor, 328
- Ambrose–Singer theorem, 316
- ancient time interval, 644
- angle, 9, 234
- angle function
 - of a path, 93
- arc length, 623
- arc-length parameter, 88
 - differential of, 88
- area, 578
- associated fiber bundle, 427
- atlas, 42

- ball
 - open, 25
- base space, 407, 424
- bases
 - equivalently oriented, 324
- Berger sphere, 423
- Betti number, 376
- Bianchi identity
 - contracted second, 261
 - first, 136, 257, 299
 - second, 257
- Bianchi map, 264
- bijection, 10

- bilinear, 77
- bilinear form, 77, 415
- binormal
 - of space path, 97
- Bishop–Gromov relative volume comparison theorem, 494
- Bishop–Gromov volume comparison theorem, 492
- blow-up point, 619
- Bochner formula, 464
 - for forms, 523
- Bolza surface, 380
- Bott periodicity theorem, 392
- bundle
 - of curvature tensors, 265
 - of Weyl curvature tensors, 266
- bundle metric, 411
- Busemann function, 500

- C^k -norm, 614
- Calabi’s trick, 508
- Cartan structure equation
 - first, 342
 - second, 344
- catenoid, 20
- Cauchy–Schwarz inequality, 9
- cellular decomposition, 354
- centrally symmetric, 602
- chart, 202
- Cheeger–Gromoll splitting theorem, 503
- Cheeger–Gromov convergence
 - pointed, 311
- Cheeger–Gromov–Taylor, 310

- injectivity radius estimate, 310
- chord-arc comparison function, 624
- chord-arc inequality, 597
- Christoffel symbols, 131, 239
- cigar soliton, 682
- classical group, 415
- closed geodesic, 283
- closed path, 87
- closed set, 26
- clutching construction, 414
- clutching map, 414
- coarea formula, 534
- cocycle condition, 407, 426
- Codazzi equation, 141, 362
- codifferential, 519
- codimension, 85
- compact-open topology, 418
- compactness theorem
 - for metrics, 311
- complete
 - vector field, 500
- complex Euclidean space, 205
- complex manifold, 205
- complex projective space, 208
- concatenated path, 255, 312
- cone, 537
- conformal
 - pointwise, 347
- conformal deformation, 643
- conformal factor, 642
- conformal Killing vector field, 685, 686
- conformal map, 347
- conformally equivalent, 349
- conformally flat
 - locally, 268
- conformally invariant, 518
- conjugate heat kernel, 513, 609
- conjugate heat operator, 512
- conjugate point, 286
- connection
 - compatible with metric, 238
 - on a principal bundle, 430
- connection 1-form, 338, 431
- connection form, 431
- continuous, 29
- contracted second Bianchi identity, 261
- contraction, 226
- coordinate chart
 - for orbifold, 389
- coordinate function, 202
- coordinate hyperplane, 10
- coordinate map, 202
- coordinates
 - geodesic, 296
- cosine function
 - modulated, 280
- cotangent bundle
 - of submanifold, 75
- cotangent space
 - of submanifold, 74
- Cotton tensor, 268
- covariant derivative
 - along a map, 243
 - Euclidean, 108, 273
 - product rule, 142
- covering map, 218
- critical point, 32, 368
 - nondegenerate, 368
- critical value, 368
- Crofton formula, 99
- crystallographic group, 273
- curvature
 - of a path, 91
 - of space path, 97
 - scalar, 261
 - sectional, 258
- curvature 2-forms, 340
- curvature form, 432
- curve, 20, 86
- curve shortening flow, 588
 - graphical, 589
 - normalized, 620
- curve shortening flow equation, 588
- cuspidal, 46
- cut locus, 303
 - in the tangent space, 303
- cut point, 302
- cutoff function, 506
- CW-complex, 353
- de Rham cohomology group, 516
- de Rham theorem, 516
- degree, 374
- derivation, 109, 213
- derivations
 - set of, 213
- derivative, 219
 - of a function, 32
- determinant, 323
- diffeomorphic, 216
- diffeomorphism, 145, 216
- diffeomorphism type, 227
- diffeomorphisms

- 1-parameter group of, 223
- differentiable, 27
 - function, 28
- differential form, 328
- directional derivative, 107
- Dirichlet eigenvalue, 529
- discrete curvature, 394
- distance, 234
 - Riemannian, 235
- distance function, 474, 502
- distribution, 147
- distributions
 - in the sense of, 174
- divergence, 162, 176
 - Euclidean, 174
- divergence theorem
 - Euclidean, 175
- double traced Gauss equation, 361
- dual metric, 337
- dual orthonormal coframe field, 337
- edge, 354
- eigenfunction
 - of Laplacian, 466
- eigenvalue
 - first, 468
 - of Laplacian, 466
- Einstein manifold, 453
- Einstein metric, 453
- Einstein tensor, 261
- embedded submanifold, 216, 358
- embedding
 - smooth, 48
 - topological, 47
- energy
 - of a path, 157
- entropy, 671
 - Hamilton's surface, 673
 - Huisken's, 611
- equatorial submanifold, 274
- equicontinuous, 613
- equivalent paths, 211
- eternal solution, 586
- Euclidean covariant derivative, 108, 273
- Euclidean heat kernel, 509
- Euclidean space
 - complex, 205
- Euler characteristic, 376
 - of orbifold, 389
- evaluation map, 78
- exotic sphere, 231
- exponential map, 250
 - for a Lie group, 420
- exterior covariant derivative, 332
- exterior derivative, 331
- fiber, 407
- fiber bundle, 424
 - fiber bundle atlas, 424
 - fiber bundle isomorphism, 424
 - fiber bundle map, 424
 - fiber space, 424
- fibration, 423
 - fibration long exact sequence of homotopy groups, 424
- figure-eight knot, 40
- finitely generated
 - group, 497
- first Bianchi identity, 136, 257, 299
- first Cartan structure equation, 342
- first derivative test, 461
- first eigenvalue, 468, 470
- first fundamental form, 101, 358
- first variation of arc-length formula, 247
- flat, 136
 - manifold, 273
- flow
 - of a vector field, 223
- foliation, 148
- forgetful functor, 228
- Four-Vertex Theorem, 99
- frame adapted to a submanifold, 358
- frame bundle, 428
- Frenet–Serret formulas, 99
- functor
 - forgetful, 228
- fundamental solution
 - to Laplace's equation, 174
- fundamental theorem of hypersurfaces, 149
- fundamental vector field, 430
- G atlas, 426
- G -equivariant, 428
- G -transition function, 426
- Gauss curvature, 120, 259
- Gauss equation, 141, 361
- Gauss lemma, 251
- Gauss map, 167
- Gauss–Bonnet theorem
 - proof of, 386
- general linear group, 415
- generator, 497
- geodesic, 129, 246

- line, 500
 - minimal, 249
 - ray, 500
 - stable, 283
- geodesic coordinates, 296
- geodesic curvature, 385
- geodesic flow, 400
- geodesic line, 249
- geodesic loop, 283
- geodesic ray, 302
- geodesic sphere, 305, 474, 481
- geodesic spherical coordinates, 477
- geodesic triangle, 393
- global solution, 586
- gradient, 220
- graph, 15
- graphical curve shortening flow, 589
- graphical map, 49
- Grassmannian, 208, 259
- Green's function, 174
- Grim Reaper, 591
- group representation, 265
- growth function, 497

- Haar measure, 422
- hairclip, 638
- half-space, 25
- Hamilton's Harnack quantity, 675
- Hamilton's surface entropy, 673
- harmonic form, 520
- harmonic function
 - on \mathbb{R}^n , 174
- harmonic map, 568
- harmonic map heat flow, 571
- harmonic mean curvature, 197
- harmonic polynomial, 468
- Harnack estimate
 - for Ricci flow on surfaces, 675
- heat equation
 - minimal fundamental solution, 512
- heat kernel
 - conjugate, 609
 - Euclidean, 509
- helicoid, 37
- Herglotz integral formula, 186
- Hessian, 31
 - covariant, 176, 345
 - Euclidean, 174
 - matrix, 31
- Hirzebruch signature theorem, 452
- Hitchin–Thorpe inequality, 454
- Hodge decomposition theorem, 521
- Hodge Laplacian, 520
- Hodge star operator, 518
- Hölder's inequality, 157
- holomorphic atlas, 348
- holomorphic chart, 348
- holomorphic map, 205, 348
- holonomy
 - Lie algebra, 313
- holonomy decomposition
 - irreducible, 317
- holonomy group, 312
 - restricted, 313
- homeomorphism, 46
- homeomorphism type, 228
- homogeneous polynomial, 467
- homotopy equivalence, 229
- homotopy lifting property, 423
- homotopy type, 229
- Hopf fibration, 218
- Hopf–Rinow theorem, 255
- horizontal lift, 399
- horizontal space, 373
- horizontal subspace, 399, 430
- Huisken's entropy, 611
- hyperbolic manifold, 279
- hyperbolic space, 275
- hyperboloid, 275
- hypersurface, 85

- immersion
 - smooth, 45
- in the barrier sense, 486
- in the sense of distributions, 484
- index
 - of a vector field, 375
- index form, 288, 570
- induced covariant derivative, 127
- induced metric, 102
- induced orientation, 325
- infinitesimal isometry, 242
- injectivity domain, 304
- injectivity radius, 309
- injectivity radius estimate, 310
 - Cheeger–Gromov–Taylor, 310
- injectivity range, 304
- inner product, 78
 - Euclidean, 7
 - of tensors, 190
 - vector space, 7
- inradius, 586
- integrable distribution, 147
- integral curve

- submanifold, 70
- integral submanifold, 147
- interior product, 329
- interior region, 165
- intersection number, 374
- interval, 86
- intrinsic, 145
- intrinsic form, 437
- inverse curve shortening flow, 582
- involutive, 147
- irreducible decomposition of R_m , 267
- irreducible representation, 265
- isolated zero, 381
- isometric, 235
- isometry, 145, 235

- Jacobi equation, 285
- Jacobi field, 284
- Jacobi identity, 110
- Jacobian
 - of the exponential map, 475
- Jordan curve theorem, 92

- k -th fundamental form, 192
- Kazdan–Warner identity, 686
- Killing vector field, 242
 - conformal, 685
- Koszul formula, 130
- Kulkarni–Nomizu product, 265

- L^2 -adjoint, 519
- L^2 -inner product, 465
- L^2 -norm, 465
- Laplacian, 176, 335, 346
 - Euclidean, 174
 - Hodge, 520
 - rough, 523
- Laplacian comparison theorem
 - local, 484
- left multiplication map, 418, 425
- left-invariant
 - metric, 420
 - vector field, 418
- Leibniz rule, 213
- length, 87, 578
 - Riemannian, 235
 - vector, 8
- level set, 32, 61
- Levi-Civita connection, 127, 237
- Lie algebra
 - associated, 419
- Lie bracket, 109
- Lie derivative, 225
- Lie group, 418
- line
 - geodesic, 500
- line bundle, 409
 - tautological, 409
- linear connection, 128, 238, 429
- linear Harnack estimate, 690
- linear hyperplane, 10
- Liouville measure, 400
- Liouville-type theorem, 505
- local section, 147
- local trivialization, 407
 - for fiber bundle, 424
- locally conformally flat, 268
- locally uniformly convex
 - hypersurface, 168
- loop, 87
- Lorentz group, 416
- lower barrier, 486

- m -th mean curvature, 195
- manifold
 - C^k , 210
 - complex, 205
 - real analytic, 203, 204
 - smooth, 203, 204
 - topological, 203, 204
 - with boundary, 208
- map energy, 568
- map energy density, 568
- map-Hessian, 568
- map-Laplacian, 568
- matrix determinant lemma, 550
- maximal rank, 35
- maximum principle, 460
 - argument, 675
- mean curvature, 119
- mean curvature flow, 574, 588
- mean curvature vector, 125
- mean value inequality, 499
- mean value property, 504
- measure zero, 306
- meridian, 134
- metric
 - induced, 102
- metric compatibility, 128
- metric dual, 237
- metric space, 9
- minimal geodesic, 249
- minimal graph, 540
 - entire, 540

- minimal hypersurface, 120
- minimal hypersurface equation for the
 - graph of a function, 541
- minimal submanifold, 534
- modulated cosine function, 280
- modulated sine function, 279
- monodromy
 - group, 313
 - representation, 313
- Morse function, 368
 - on manifold with boundary, 387
- Morse index, 368
- Morse inequalities, 379
- moving frame, 336
- Myers's theorem, 292

- Neumann eigenvalue, 529
- Newtonian potential, 174
- nondegenerate bilinear form, 415
- nondegenerate critical point, 368
- nonorientable, 325
- nonorientable vector bundle, 411
- norm, 234
- normal bundle, 73
- normal space, 73
- normal vector, 10
- normalized curve shortening flow, 620
- normalized Ricci flow on surfaces, 645
- north pole, 14
- null-eigenvector condition, 575
- null-homotopic, 313

- open set
 - in \mathbb{R}^n , 25
 - in metric space, 27
- orbifold, 388
- order
 - of a singular point, 388
- orientable manifold, 325
- orientable vector bundle, 411
- orientation, 324
 - class, 324
 - induced, 325
 - of a manifold, 324
 - on a vector bundle, 410
 - opposite, 324, 325
- orientation number, 374
- orientation-preserving map, 326
- orientation-reversing map, 326
- oriented hypersurface, 165
- oriented manifold, 325
- origin
 - in Euclidean space, 5
- orthogonal group, 416
- orthonormal, 102
- orthonormal coframe field
 - dual, 337
- orthonormal frame bundle, 429
- orthonormal frame field, 336
- outradius, 586

- paperclip, 638
- parabolic rescaling, 633
- parallel, 128, 143, 240, 245
 - hyperplanes, 12
- parallel transport, 245
- parallel vector subspace, 51
- parallelizable manifold, 410
- parametrization, 37
- parametrized by arc length, 89
- parametrized submanifold, 37
- path, 86
 - closed, 87
 - periodic, 87
 - reversed parametrization, 88
- paths
 - equivalent, 211
- Pfaffian curvature form, 437
- plane bundle, 409
- Plateau problem, 560
- Poincaré ball, 275
- Poincaré–Hopf theorem, 376
- Poincaré lemma, 516
- Poisson equation, 269
- polyhedral metric, 394
- positively oriented basis, 325
- potential function, 656
- primitive 2-form, 260
- principal bundle, 428
 - isomorphism, 428
 - smooth, 428
- principal curvature, 117
- principal direction, 117
- principal unit normal
 - of space path, 97
- product fiber bundle, 425
- product rule
 - for Laplacian, 529
- projection map, 68, 407, 424
- projective space
 - complex, 208
 - real, 207
- proper map, 49
- properly embedded submanifold, 216

- pseudosphere, 155
- pull-back, 224, 235, 330
 - of a tensor, 81
- pull-back vector bundle, 413
- push-forward, 224

- quasilinear elliptic equation, 541
- quotient set, 18
- quotient space, 27

- radial graph, 19
- radial projection, 19
- Rauch comparison theorem, 294
- ray, 302
 - geodesic, 500
- Rayleigh principle, 470
- Rayleigh–Ritz quotient, 470
- reaction-diffusion equation, 644
- real analytic function, 31
- real analytic manifold, 203
- real projective space, 207
- regular value, 362, 373
- relative entropy, 671
- relatively open subset, 58
- reparametrization, 89
- representation
 - group, 265
 - irreducible, 265
- rescaling, 632
- Riccatti equation, 479
- Ricci flow
 - on surfaces, 644
- Ricci soliton
 - shrinking, 681
 - steady, 682
- Ricci tensor, 260
- Riemann curvature 4-tensor, 142
- Riemann curvature tensor, 256
 - of submanifold, 135
- Riemann surface, 348
- Riemannian manifold, 102, 234
- Riemannian metric, 102, 233
 - components, 103
- right multiplication map, 428
- right-invariant
 - metric, 420
- rough Laplacian, 523
- ruled surface, 37

- Sard’s theorem, 374
- Sasaki metric, 399, 400
- scalar curvature, 261

- Schouten tensor, 267
- second Bianchi identity, 257
- second Cartan structure equation, 344
- second derivative test, 460
- second fundamental form, 115, 125, 358
- second variation of arc-length formula, 282
- section
 - of a fiber bundle, 424
 - of a vector bundle, 413
 - of tangent bundle, 398
 - smooth, 414
- sectional curvature, 258
 - radial, 365
 - spherical, 365
- self-adjoint
 - formally, 465
- self-similar solution, 510
 - shrinking, 589, 611
 - translating, 592
- semicubical parabola, 46
- shrinking Ricci soliton, 681
- shrinking self-similar solution, 589, 611
- Sierpiński Carpet, 26
- Simons’s inequalities, 556
- simple closed curve, 560
- sine function
 - modulated, 279
- singular point, 388
- singular solution, 618
- singularity
 - Type I, 618
 - Type II, 618
- singularity model, 621
- singularity time, 618
- smooth
 - embedding, 48
 - function, 31
 - immersion, 45
- smooth embedding, 37, 215, 358
- smooth manifold, 203
- Sobolev norm, 468
- Sobolev space, 468
- space curve, 97
- space form
 - simply connected, 279
- space path, 97
- spacetime rescaling, 619
- special linear group, 417
- special orthogonal group, 417
- special unitary group, 417

- speed, 89
- sphere, 12
 - distance, 481
- spherical space form, 274
- splitting theorem
 - Cheeger–Gromoll, 503
- stable geodesic, 283
- stable homotopy group, 392
- stable minimal hypersurface, 565
- stable minimal submanifold, 556
- starshaped, 19, 516
- steady Ricci soliton, 682
- stereographic projection, 14
- strictly convex
 - hypersurface, 168
- structure constants
 - for Lie group, 422
- structure equations
 - Cartan, 342
- structure group, 426
 - reduction, 427
- subatlas, 203
- sublevel set, 377
- submanifold, 49
 - parametrized, 37
- subrepresentation, 265
- subspace topology, 27
- supersolution
 - to heat equation, 651
- support function, 170
- surface, 20
 - of revolution, 20
- surface entropy monotonicity formula, 673
- symplectic group, 416
- tangent bundle, 221
 - submanifold, 68
- tangent plane
 - submanifold, 58
- tangent space, 211
 - submanifold, 58
- tangent vector, 88, 211
 - coordinate, 212
 - unit, 88
- tautological line bundle, 409
- tautological vector bundle, 409
- tensor bundle, 82
- tensor contraction, 78
- tensor product, 78, 105
- tensor product bundle, 412
- tensor space, 82
- tensorial form, 432
- top-degree form, 328
- topological embedding, 47
- topological group, 417
- topological manifold, 203
 - underlying, 204
- topological space, 27
- torsion
 - of space path, 98
- torsion-free, 111, 128, 238
- torus, 20
- total curvature
 - of a path, 95
- total derivative, 28
- total Gauss curvature, 379
- total space, 407, 424
- totally umbilical, 162, 481
- traced Gauss equation, 361
- tractrix, 155
- transition function, 407
- transition map, 42, 54
- translating self-similar solution, 592
- trefoil knot, 40
- triad, 387
- triangle, 354
- triangulation, 354
- trivial bundle, 74
- trivial fiber bundle, 425
- trivial vector bundle, 410
- Type I singularity, 618
- Type II singularity, 618
- unimodular Lie group, 422
- unit normal field
 - along a path, 90
- unit normal vector, 72
 - inward, 73
 - outward, 73
- unit sphere bundle, 425
- unit tangent bundle, 400
- unitary group, 416
- unknot, 40
- upper barrier, 486
- upper half-space, 275
- variation vector field, 247
- vector bundle, 407
 - homomorphism, 412
 - isomorphism, 413
 - nonorientable, 411
 - orientable, 411
 - over submanifold, 74

- pull-back, 413
- tautological, 409
- trivial, 410
- vector field
 - along a map, 242
 - along a path, 88
 - associated to group of diffeomorphisms, 223
 - complete, 500
 - conformal, 685
 - submanifold, 69
- vector space, 6
- vertex, 354
- vertical lift, 399
- vertical space, 430
- vertical subspace, 399
- volume form, 333

- wallpaper group, 273
- weakly convex
 - hypersurface, 168
- weakly stable
 - harmonic map, 570
- wedge product, 329
- Weingarten map, 114
- Weyl tensor, 263
- Weyl-Schouten tensor, 267
- Whitney sum, 74, 412
- Willmore conjecture, 191
- Willmore energy, 191
- winding number
 - of a path, 96

- Yamabe problem, 567
- Yau's gradient estimate, 505

- zero-section
 - of a vector bundle, 413