
Index

Notation is indexed under the initial letter or abbreviation; Greek letters appear in spelled-out order.

In certain entries a boldface page number indicates where the term is defined; however, the use of that convention is not systematic.

- ! superscript ($f^!$), 276
- $|\mathcal{L}|, |D|$, **18**
- //, **155**
- * subscript (f_*), 276
- * superscript (f^*), 276
- 1-dimensional ring, 69
- 1-generic, 292, 293
 - 2×2 matrices, classification of, **311**
- $2r + 3$ lemma, 296
- 2-sheeted cover
 - of \mathbb{P}^1 , 109, 112, 149
 - of M_4 , 178
 - over a genus-1 curve, 219
 - unramified, 129
- 3-sheeted cover, 117, 120, 122, 129
 - of \mathbb{P}^1 , 172
 - of conic, 219
- 4-sheeted cover
 - of \mathbb{P}^1 , 175

- Abel, Niels, 364
- Abel's theorem, 101, 104
- Abel–Jacobi map, 100, 102, 105, 109, 205
 - differential, 109
- abelian algebraic group, 83
- abelian function, 368
- ACM, 59, 69, 73, 79, 89, 179, 184, 187, 269,
306, 316, 321, 333, 345
 - variety, 69

- Acosta Jaramillo, Enrique, 67
- acyclic, **315**, 332
 - EN complex, 323
- adjoint ideal, 264, 266
- adjoint pair of functors, 276
- adjoint scheme, 259
- adjoint series, completeness of, 261
- adjunction, 288
- adjunction formula, 33, 49, 77, 85, 109, 193,
194, 254, 272, 339, 348, 385
- admissible cover, 161
- affine scheme quotient, 95
- al-Khāyāmī, Umar ibn Ibrāhīm, 355
- algebraic geometry, founding of, 370
- algebraic group, projective, 97
- Alper, Jarod, 157
- ample, 23, 321, *see also* very ample
- analytic geometry, 359
- Anglade, Marie, 356
- Annales de Mathématiques Pures et Appliquées*, 358
- anticanonical sheaf, 218
- Apollonius, 354
- arithmetic genus, **39**, 42, 54, 150, 161, 183,
220, 274, 287
- arithmetically Cohen–Macaulay, *see* ACM
- arithmetically Gorenstein, 316, 317, 332
 - but not a complete intersection, 333
- ascending double link, 280

- associated prime, 69
 Auslander, Maurice, 310
 Auslander–Buchsbaum formula, 71, 310, 322, 326
 automorphism, 84, 95, 157
 –s are rare if $g > 2$, 230
 group, 85, *see also* under PGL
 group, size of, 228, 385
 Azumaya algebra, 58

 Bacharach, Isaak, 370
 Baltzer, Richard, 370
 barycentric coordinates, 367
 base change, 149
 base change theorem, 141
 base locus, 9, 19, 77
 basepoint free, 298, 299, 303, 304
 basic double link, 273
 basis theorem, *see under* Hilbert
 Bass, Hyman, 281
 Bayer, Dave, 313
 Beauville, Arnaud, 218
 Bertini, Eugenio, 291, 372
 Bertini irreducibility theorem, 206
 Bertini’s theorem, 9, 86, 87, 122, 168, 175, 181, 303, 380
 Betti numbers, 310
 Betti table, 313, 317, 323, 326, 329, 330, 333
 Bézout, Étienne, 363
 Bézout’s theorem, 9, 86, 119, 123, 168, 173, 217, 271, 272, 339, 365, 372, 380
 elementary, 176
 bidegree, 112
 bielliptic, 220, 221
 bilinear form, 125
 birational, 172, 177, 183, 192
 morphism, 109
 to a nodal plane curve, 189
 transformation, 369
 birationally very ample, 23, 205, 209
 bitangent, 123, 365
 to a plane quartic, 2, 123, 125, 129
 blowdown, 51, 301, 304
 blowup, 88, 258, 299, 371, 381, 383
 of $J(C)$, 106
 of a cone, 290
 of a smooth surface, 50
 Bombelli, Rafael, 355
 boundary, 156
 branch divisor, 34, 111
 branch point, 109
 simple, 85, 116, 129

 branched cover of \mathbb{P}^1 , 115
 Briend Jean-Yves, 356
 Brill, Alexander, 1, 76, 80, 209, 269, 369, 372
 Brill–Noether number, 348
 adjusted, 250
 Brill–Noether theorem, 158, 161, 348
 for curves of given gonality, 215
 in low codimension, 343
 with inflection, 250
 Brill–Noether theory, 158, 209–215
 Brouillon Project, 356
 Buchsbaum, David A., xv, 310
 Buchweitz, Ragnar-Olaf, 228, 239
 bundle, *see under* line, vector
 Burch, Lindsay, 321

 C_d^r , 105, 191, 211
 Calabri, Alberto, 225
 calculus, 98
 canonical curve, 45, 189, 191, 255
 of genus 4, 167, 171, 341
 of genus 5, 176
 of genus 6, 217
 canonical divisor, 272
 canonical embedding, 103, 385
 canonical ideal, 262
 canonical linear series, *see under* linear series
 canonical model, 121, 384
 canonical module, cyclic, 281
 canonical morphism, 43
 canonical sheaf, 31
 of \mathbb{P}^r , 31
 square root of, 123
 Cardano, Gerolamo, 355
 Cartier divisor, 14, 16, 30, 49, 52, 94, 253, 256, 275, 288, 299
 Castelnuovo, Guido, 58, 179, 241, 295, 370
 Castelnuovo bound, 184, 300
 Castelnuovo curve, 184, 195, 306, 307, 384
 Castelnuovo’s theorem, 50, 179, 183, 209, 351
 Catalan number, 213
 Cauchy, Augustin-Louis, 358, 363, 366, 368
 Cauchy integral theorem, 367
 Cayley, Arthur, 311, 366, 368
 Cayley–Bacharach–Macaulay theorem, 46, 80
 center of a projection, 22
 del Centina, Andrea, 353, 357, 367, 368
 Chasles, Michel, 357, 358

- Chern class, first, 284
 Chow ring, 234, 242
 Clebsch, Rudolf Friedrich Alfred, 11, 76, 269, 369, 370
 Cliff \mathcal{L} , 329
 Clifford index, 328
 Clifford's theorem, 47, 106, 121, 128, 169, 183, 209, 230, 329
 equality in, 191
 closure theorem (Poncelet), 358
 clusters of points, 76
 coarse moduli space, 153
 codifferential, 103
 Cohen–Macaulay, 59, 68, 76, 326, *see also*
 ACM
 module, 316
 ring, 59, 314
 scheme, 70, 316
 coherent sheaf, 15
 cohomology and base change, 149
 compactification
 Deligne–Mumford (\overline{M}_g) , 154, 156
 modular, 154, 151–156
 of the affine line, 155
 Satake, 154
 complete flag, 232
 complete intersection, 69, 73, 146, 158, 217, 219, 269, 272, 285, 286, 317, 345
 local, 277
 of quadric and cubic in \mathbb{P}^3 , 168
 of three quadrics, 173, 175
 of two quadrics, 86, 271, 287
 of two surfaces in \mathbb{P}^3 , 278
 complete invariant of linkage, *see*
 Hartshorne–Rao module
 complete linear series, 19, 79, 266, 292, 298
 complete quadrilateral, 356, 357
 completeness of the adjoint series, 261
 complex analytic viewpoint, 40
 complex functions, 368, 370
 complex manifold, 99, *see also* Riemann
 surface
 complex numbers, 8, 356, 358
 complex torus, 100
 conditions imposed on a linear series, 25
 conditions of adjunction, 254
 conductor, 247, 259, 261, 262, 282
 cone
 over a quadric surface, 175
 over elliptic quintic, 219
 quadratic, 169, 171
 conic, 20, 58
 pole and polar, 357
 projective definition, 358
 tangent to, 355
 conic section, 354
 theory of $-s$, 355
 conics, theory of, 357
 connected in codimension 1, 71
 conormal sheaf, 138
 contour integration, 364
 coordinate ring, 108
 homogeneous, 59
 coordinate system, 355, 360
 coordinates system, 360
 Copley medal, 366
 Coşkun, İzzet, xv, 339
 cotangent bundle, 31, 78
 cotangent map, 102
 cotangent space, 102, 103
 to $J(C)$, 101
 counit, 276
 counting maps between curves, 144
 cover
 2-sheeted, 85, 88
 of \mathbb{P}^1 , 87
 3-sheeted, 167, 168
 covering space, 115
 Cramer, Gabriel, 172, 362
 Cramer's rule, 320
 Crelle, August Leopold, 358
 Cremona, Luigi, 371
 Cremona transformation, 122, 370, 371
 crimping, 244
 cross-ratio, 356, 358
 cubic curve, 20, 364, *see also* twisted cubic
 classification of $-s$, 361
 going through nine points, 363
 plane, 85, 146, 155, 207, 362
 cubic equation, geometric solution, 355
 cubic form, 85
 cubic surface
 in \mathbb{P}^3 , 218
 in \mathbb{P}^4 , 178
 cup product pairing, 48
 curve
 Castelnuovo, 316
 convention on, 8
 rational normal of degree d , 20
 with level m structure, 157
 curvilinear, 105, 383
 cusp, 43, 264, 365, 369

- on quartic, 118
 - ramphoid, 172
- cuspidal curve
 - g -, 245
 - ideal sheaf on, 247
 - of genus 1, 151
 - Picard variety of, 245
- cyclic canonical module, 281
- cycloid, 361
- Darmstadt Polytechnic, 370
- Dedekind complementary module, 262
- deficiency module, 270
- deformation bound, 342
- deformation theory, 245
- deformation-rigid curve, 345
- degree
 - of a divisor, 13
 - of a projective scheme, 52
 - of a sheaf, 284
 - of morphism, 15
 - of projective subscheme, 52
- del Pezzo, Pasquale, 291
- del Pezzo surface, 218
 - of degree 3, 218
 - of degree 5, 89, 217–219
- Deligne, Pierre, 4
- Deligne–Mumford compactification, 154
- Δ (Cartier divisor), 260
- δ invariant, 43, 55, 118, 194, 265
 - of ordinary multiple point, 265
- depth
 - lemma, 332
 - of a module, 315
 - of an ideal, 68
- Desargues, Girard, 356
- Desargues theorem, 357
- Descartes, René, 1, 356, 360
- descriptive geometry, 369
- desingularization, 371
- differential
 - regular, 254
- differential form, 368
 - on nodal plane curve, 254
 - regular, 77
- dimension of linear series, 18
- dimensionality, 360
- dimensionally transverse, 232
- Diocles, 354
- diophantine problems, 360
- directly linked, 269
- directrix, 290
- Dirichlet principle, 76, 370
- discriminant
 - hypersurface, 51, 160, 162, 175
- disjoint lines, arithmetic genus of, 54
- $\text{Div}(C)$, $\text{Div}_0(C)$, 13
- div, 30
- divisible group, 83
- divisor, 12, *see also* Cartier, effective, Weil
 - associated to a function, 13
 - class group, 156, 299
 - inflectionary, 224, 226
 - on curve, 12
 - principal, 13
 - special, 46
- dominant map, 158, 159
 - between varieties of same dimension, 198
- double curve, *see* cover, 2-sheeted
- double line on a quadric, 71
- double links, 286
- double plane, 51
- double point, *see* node
- double pole, 77
- doubling the cube, 354
- doubly transitive, 202
- dual curve, 364
- dual line, 170
- duality, 37, 170, 274, 357, 359, *see also* Serre
 - paradox, 364
- dualizing module, 263, 277
- dualizing sheaf, 31, 270, 274, 275
 - for singular curves, 284
- Dyck, Walther, 370
- Eagon–Northcott complex, 176, 314, 317, 322, 387
- École Polytechnique, 358
- effective divisor, 12, 13, 79, 93, 256
 - family of $-s$ on a curve, 135
- Eisenbud, Monika, xv
- elementary Bézout theorem, 176
- elimination theory, 9
- Eliot, Thomas Stearns, 289
- ellipse, 355
- elliptic
 - curve, 83, 166, *see also* genus 1
 - naming, 99
 - integral, 98
 - normal curve, 239
 - quartic, 87
 - quintic, 88
- $EN(\phi)$, 317

- endomorphism ring, 249
 Enriques, Federico, 370
 enumerative geometry, 134
 $\epsilon = \epsilon(d, r)$, **183**, 195
 equivalence, *see under* linear
 Erlangen University, 359, 370
 Euclid, 354, 355
 Eudoxus, 354
 Euler, Leonhard, 172, 362–364
 Euler characteristic, 29, 49, 50
 Euler sequence, 32
 Euler's formula, 226
 Eutocius, 354
 evenly linked, **273**
 exact complex, 310
 expected dimension, 164, 341
 Ext_S^m , 315
 exterior power, 33, 314
 extraneous components, 337
 extremal number of quadrics, 61

 $\mathfrak{F}(C_0)$, 255, 259
 \mathfrak{f}_{C/C_0} , 259
 family, 135, *see also* pencil
 of curves, 96
 of divisors, 19
 of effective divisors, 94
 of invertible sheaves, 96, 98
 universal, 96
 fat point, 14, **383**
 Fermat, Pierre de, 360
 Ferrand, Daniel, 278
 Ferrari, Ludovico, 355
 fiber of invertible sheaf, 17
 fine moduli space, 94, 96, 105, 108, 136
 finite group action, 108
 finite map, 95, 276
 finite type, 136, 145
 Fitting, Hans, 314
 Fitting ideal, 314
 flat family, 136, 138
 of projective schemes, 140
 of subschemes, 137
 flat limit, 287
 flat map, 71
 flat module, 139
 flex, 2, 200, 207, 362, 365
 form of \mathbb{P}^1 , 58
 fractional exponent, 366
 free action, 108
 free group, 115
 free resolution, 139, **310**
 minimal, 310
 Friedelmeyer, Jean-Pierre, 367
 Frobenius map, 35
 Fulton, William, 212
 Fulton's elementary Bézout theorem, 176
 function field, 198
 fundamental class, 48
 fundamental group, 115
 fundamental theorem
 (Noether), 80, 364, 372, 373
 of algebra, 1, 364
 of calculus, 103

 $G(2, V)$, 339
 $G(k, V)$, $G(1, V) = \mathbb{P}(V^*)$, $\mathbb{G}(k, r)$, **8**
 $G(r + 1, d + 1)$, 231
 $\mathbb{G}(1, 3)$, 126, 130, 134
 $\mathbb{G}(k, r)$, **142**
 g-fold point, 105
 g + 1 theorem, 109, 160
 g + 2 theorem, 104, **192**, 194
 g + 3 theorem, 103, 177, 178
 g_2^1 , 300
 g_3^1 , 172, 300
 g_4^1 , 172, 175, 216, 252
 g_d^1 , 173, 266, 317
 g_5^2 , 220
 g_6^2 , 216
 g_d^r , **19**, 209, 231, 241
 Galois normalization, 198
 Galois theory, 198, 263
 Gauss, Carl Friedrich, 1, 57, 364
 Gauss map, 103, 170
 of a quadric, 171
 a general X has property Y, 66
 general n -pointed curve, 250
 general curve of genus g , 67
 general divisor, 109
 general hyperplane section, 179
 general invertible sheaf, 67
 general type, 159
 generalized row, 293
 generators of homogeneous ideal, 214
 generically finite, 109
 genus, **39**, 369, 371, *see also* arithmetic,
 geometric, and the numbers below
 invariance under birational maps, 372
 genus 0 curve, 57–58
 genus 1 curve, 75–89
 classification of $-s$, 148
 moduli space of, 147
 genus 2 curve

- dimension of space of $-s$, 120
 embeddings in \mathbb{P}^3 , 119
 maps to \mathbb{P}^1 , 117
 maps to \mathbb{P}^2 , 118
 genus 3 curve, 121–125, 227, 252
 genus 4 curve, 167–172, 177
 genus 5 curve, 172–173
 canonical, 173–177
 geometric genus, 39, 40, 42, 161
 geometric invariant theory, 95, 153
 geometric Riemann–Roch theorem, 45, 107
 Gergonne, Joseph Diaz, 358, 364
 germ, 283
 Geschlecht, 11
 Gieseker, David, 212
 Giessen University, 369, 370
 Gilbert, William Schwenck, 3
 global section, 83, 223
 of coherent sheaf, 15
 value at a point, 16
 gonality, 55, 215
 of nodal plane curve, 265
 good projection, 189
 Gordan, Paul, 369
 Gorenstein, Daniel, 281
 Gorenstein, 76, 247, 273, 277
 locally, 46
 module, 316
 Gorenstein scheme, 31
 Göttingen University, 369, 370
 Gotzmann, Gerd, 142
 grade, 69, 319, 323
 of an ideal, 68, 314
 graded module, 309
 Grassmannian, 8, 66, 68, 87, 125, 126, 130, 142, 203, 233, 234, 238, *see also* G , \mathbb{G}
 Greek mathematics, 354
 Green, Mark, 326, 330
 Green’s conjecture, 326, 330
 Griffiths, Phillip A., xv, 209
 Grothendieck, Alexander, 1, 4, 93, 145, 276, 281
 convention on projective space, 8
 group action, 95
 group law, 80, 266
 Gruson, Laurent, 189

 $H_*^i(\mathcal{F})$, 10
 $h(g, r, d)$, 164
 $h^i(\mathcal{F})$, $h^i(D)$, 10
 $\mathcal{H}_{g,r,d}^\circ$, 345
 $\mathcal{H}_{g,3,d}$, 335, 338, 341–345
 $\mathcal{H}_{0,r,d}$, 338
 $\mathcal{H}_{0,3,3}$, second component of, 335
 $\mathcal{H}_{0,3,3}^\circ$, 349
 $\mathcal{H}_{0,3,4}$, 338
 $\mathcal{H}_{0,3,5}$, 339
 $\mathcal{H}_{1,3,4}$, 339
 $\mathcal{H}_{1,3,4}^\circ$, 350
 $\mathcal{H}_{2,3,5}$, 340
 $\mathcal{H}_{2,3,5}^\circ$, 350
 $\mathcal{H}_{3,3,6}^\circ$, 351
 $\mathcal{H}_{4,3,6}$, 341
 $\mathcal{H}_{9,3,8}^\circ$, 346
 $\mathcal{H}_{10,3,9}^\circ$, 347
 $\mathcal{H}_{14,3,24}^\circ$, 348
 \mathcal{H}_{ci}° , 344, 352
 $\mathcal{H}_d(\mathbb{P}^r)$, 337
 Halmos, Paul, 3
 Halphen, Georges-Henri, 1, 269
 Hamilton, William Rowan, 364
 Hartshorne, Robin, 269
 Hartshorne–Rao module, 270, 280, 285, 286, 332
 minimal, 281
 Harvard University, xv
 Heidelberg University, 370
 Hesse, Ludwig Otto, 365, 367
 Hessian, 90, 226, 365
 Hilb_{3m+1} , 349
 Hilb_{dm-g+1} , 335, 337
 $\text{Hilb}_{p(m)}$, 137
 Hilbert, David, 311, 321
 Hilbert function, 39, 146, 184, 189, 195, 286, 310, 317
 Hilbert polynomial, 53, 146, 364
 Hilbert scheme, 93, 136–146, 153, 270, 335–352
 at a point, 138
 restricted, 335
 Hilbert’s basis theorem, 310
 Hilbert’s syzygy theorem, 310
 Hilbert–Burch theorem, 123, 321
 Hippocrates of Chios, 354
 Hire, Philippe de la, 357
 historical context, 1–2, 57, 93, 98, 134, 161, 162, 182, 209, 241, 269, 310, 311, 353–373
 Hodge index theorem, 228, 385
 Hodge theory, 99
 holomorphic differential, 99, 372

- homogeneous coordinate ring, 9, 59, 60, 189
 homogeneous equation, 367
 homotopy theory, 198
 $\text{Hur}_{g,d}^{\circ}$, 159
 Hurwitz, Adolf, 161
 Hurwitz number, 117, 161
 Hurwitz space, 135, 159–161
 Hurwitz's theorem, 34, 35, 37, 57, 111, 113, 145, 276, 381
 hyperbola, 354
 hyperelliptic curve, 44, 46, 55, 88, 103, 104, 106, 111, 191, 194, 229, 238, 305, 344, *see also* 2-sheeted cover, involution determined by $2g + 2$ points, 112 Weierstrass points, 227
 hyperelliptic-trigonal dichotomy for curves of genus ≤ 4 , 169
 hyperflex, 227
 hyperplane section, 61
 I_t , 314
 Iarrobino, Anthony A., Jr., 337, 351
 ideal, *see also* Fitting of minors, 243 scheme of left $-s$, 58 sheaf, 247, 387
 imaginary numbers, naming of, 364
 immersion, 224
 incidence correspondence, 158, 339
 independent conditions, 62, 81, 163, 174, 175, 184, 255, 382, 386
 indeterminacy locus, 22
 infinitely near point, 259 double, triple, 266
 infinitesimal motion of a curve, 138
 infinity, line/point at, 363, 367
 inflection point, 223, *see also* flex finiteness of, 224
 inflectionary, *see also* divisor behavior, 231, 250
 inner automorphism, 115
 integral endomorphism ring, 249
 integral theorem (Cauchy), 367
 integration, 98, 99
 interpolation of $d + 3$ points, 63
 intersection multiplicity, 48
 intersection number, 285
 intersection pairing, 48
 intersection theory, 388
 invariance under group action, 95
 invariant form, 165 of symmetric group, 95 ring of $-s$, 154
 invertible sheaf, 15, 16, 93, 223 characterization, 18 family of $-s$ on a curve, 135 isomorphism between $-s$, 30
 involution, hyperelliptic, 113
 Islamic mathematics, 355
 Isom, 146, 150
 isotropic subspace, 125
 $J(C)$, 100
 j -function, 148, 165
 $\text{Jac}(C)$, 93
 Jacobi, Carl Gustav Jacob, 364
 Jacobi inversion theorem, 102
 Jacobian, 88, 100, 135 analytic construction, 98 points of order 2, 129, 130
 Jordan, Camille, 198
Journal de Mathématiques Pures et Appliquées, 358
Journal für die reine und angewandte Mathematik, 358, 366
 K_C , 31
 K_X, K_X^* , 14
 κ , 263
 $\kappa(p) := \mathcal{O}_{X,p}/\mathfrak{m}_{X,p}$, 17
 Kelly, Leroy Milton, 91
 Kempf, George, 209, 242
 Kharlamov, Viatcheslav, 225
 Khayyām, Omar, 355
 Kleiman, Steven L., 209, 242
 Klein, Felix, 359, 369, 370
 Klein 4-group, 148
 Koszul complex, 32, 311, 313, 317, 318, 320, 321, 324
 Kronecker, Leopold, 182, 311, 312, 369
 Krull dimension, 8, 310
 Kulikov, Viktor S., 225
 Kummer, Ernst, 369
 Künneth formula, 52, 387
 \mathcal{L}_p , 17
 Laksov, Dan, 209, 242
 λ -line, 150
 Larson, Eric, 214
 Lasker's theorem, 8, 168, 173, 339
 lattice, 99, 100
 Lazarsfeld, Robert, 212

- Lazarsfeld–Rao property, 280
 Lefschetz fixed point formula, 228
 left ideals, scheme of, 58
 Leibniz, Gottfried Wilhelm, 357
 Leray spectral sequence, 43, 52, 258, 306
 level structure, 157
 Levy, Silvio, xv
 liaison, *see* linkage
 licci, 270
 limit of invertible sheaves, 247
 Lindemann, Ferdinand, 370
 line bundle, 17
 line geometry, 366
 linear dependence, 363
 linear equation, system of $-s$, 363
 linear equivalence, 13, 79, 272
 of divisors, 76
 linear series, 18, 19, 364, *see also* complete
 canonical, 113, 226
 cut out by curves, 253
 cut out on X by V , 216
 of degree 4, 86
 on \mathbb{P}^1 , inflections of, 231, 250
 on nodal plane curves, 256
 sums of, 205
 tricanonical, 153
 linear system, *see* linear series
 linearization, 154
 linearly equivalent, *see* linear equivalence
 linearly general position, 63, 179, 185
 linearly normal, 22, 59, 69
 linearly reductive group, 155
 linkage, 270–288, 361
 addition, 286
 linked, evenly linked, 273
 Liouville, Joseph, 358
 Littlewood–Richardson rule, 234
 local ring, module over, 309
 locally Cohen–Macaulay, 278
 locally complete intersection, 245
 locally free sheaf, 15, 386
 locally Gorenstein, 46
 Lüroth, Jacob, 57
 Lüroth’s theorem, 58, 148

 $M = M(d, r)$, 183
 M_4 , 2-sheeted cover of, 178
 M_g, \overline{M}_g , 147, 153, 154, 157
 dimension, 160
 irreducibility, 161
 $\mathfrak{m}_{X,p}$, 17
 Möbius, August, 367

 Macaulay, Francis Sowerby, 1, 76, 80, 269,
 363, 372
 Macaulay (software), 313
 MacLaurin, Colin, 362, 365
 Magnus, Leopold, 371
 main theorem of elimination theory, 9
 Manin, Yuri I., 218
 Maroni invariant, 307
 Martens’ theorem, 190
 extensions, 108
 matrix pencil, 312
 Matsusaka, Teruhisa, 141
 Matsusaka’s theorem, 141, 143, 144, 146
 Maurolico, Francesco, 356
 maximal M -regular sequence, 315
 maximal rank, 214
 maximal rank theorem, 214
 maximal regular sequence, 69
 maximally special curve, 344
 Mazur, Barry, 102
 Menaechmus, 354
 meromorphic
 differential, 98
 function, 99
 Mesopotamia, 353
 minimal degree, 291
 minimal free resolution, 310, 316
 minors
 2×2 , 297
 minors, 2×2 , 59
 modular compactification, *see under*
 compactification
 module
 over a local ring, 309
 moduli functor, 135
 moduli problem, 134, 136
 moduli space, 83, 84
 fine, 94, 136
 of genus 1 curves, 147
 moduli-rigid curve, 345
 Monge, Gaspard, 358
 monodromy, 161, 340
 monodromy group, 197, 197–207
 transitive, 198
 morphism
 canonical, 43
 degree of, 15
 μ, μ_d , 100
 mulsec, 180
 mult $_p$, 48
 multiple point, 50, 105, 363

- multiplication map, 292
multiplicity, 53, 55
 of an intersection, 363
 of divisor, 12
multisecant, 180, 190
Mumford, David, xv, 4, 125, 153, 348
Munich Polytechnic, 370
mystical hexagram, 357

Néron–Severi group, 228
Nakayama’s lemma, 249, 309, 310, 324, 388
Nasu, Hirokazu, 348
natural
 correspondence, 136
 transformation, 108
nef but not necessarily ample, 218
net, 21
Newton, Isaac, 361
nodal curve, 75, 189, 206, 252, 253, 266
 differential on, 254
 gonality, 266
node, 43, 104, 283, 365
 condition imposed by, 163
 on quartic, 118
Noether, Max, 1, 76, 80, 209, 269, 363, 369, 371
Noether normalization, 275
nonhyperelliptic curve, 230
 of genus 3 and degree 6, 322
 of genus 4, 177
nonspecial, 212
nonzerodivisor, 314, 321, 322
normal
 bundle, 73, 335
 of rational normal curve, 73
 crossing, 257
 linearly / quadratically / n -ically, 59
 ring, 69
 scheme, 14
 sheaf, 138
 to a conic, 355
 to an algebraic curve, 361
normalization, 42, 253, 256, 257, 259, 260, 262, 265–267
number field, 198
number of conditions, *see* independent conditions
numerical equivalence, 228
numerical uniform position lemma, 204

octic curve, 178
 in \mathbb{P}^4 , 173
 ω_{R_X} , 316
orbit, 95
 $\text{ord}_p(f)$, order of a function, 13
ordinary n -fold point, 264
ordinary multiple points, 257
ordinary singularity, 50
orientation, 320
osculating flag, 235
osculating space, 238
Osserman, Robert, 4

 $\mathbb{P}(V)$, \mathbb{P}^r , 8
 $\mathbb{P}^1 \times \mathbb{P}^1$, *see* bidegree
 \mathbb{P}^3 , 168
 any 4 curves intersect a common line, 134
 \mathcal{P} -function, 99
Pappus problem, 361
parabola, 354, 363
paradox
 about dual curves, 364
 Cramer’s, 362
parallel postulate, 355
parameter space, 105
parametrization of the space of curves, 84
partial normalization, 248, 386
Pascal, Blaise, 357
Pascal’s theorem, 357
path integral, 98, 99
pd, 310
pencil, 21
 etymology, 21
 of curves, 364
 of quadrics, 86, 87
perfect ideal, 76
perfect pairing, 274
permutation, *see* symmetric group
Perron, Oskar, 182
Peskine, Christian, 189, 269
Petri’s theorem, 328
Pfaffian, 89, 177
Pflueger, Nathan, 228
 PGL_2 , 85, 120, 147, 338, 344, 384
 PGL_3 , 85, 120, 155
 PGL_4 , 141, 337, 384
 $\text{PGL}_4 / \text{PGL}_2$, 272
 PGL_{5g-5} , 153
 PGL_{r+1} , 382
 π transcendental, 370
 $\pi(r, d)$, 300

- $\pi_1(d, 3)$, 189
 $\text{Pic}_0(C)$, 93
 Pic_d , 97
 $\text{Pic } C$, 13
 Picard group, 13, 51, 84, 124, 161, 301
 Picard variety, 135, 153, 243, 246
 relative, 94, 243, 246
 Picard variety/scheme, 93, 94
 Piene, Ragni, 337
 Pieri's formula, 234
Pirates of Penzance, 3
 Plücker coordinates, 130, 143
 Plücker embedding, 126, 130, 142, 233
 Plücker equation, 131
 Plücker formula, 224, 239
 Plücker relation, 143
 Plücker, Julius, 364, 365, 370
 plan and elevation, 358
 plane curve, 75, 137, *see also* uder cubic,
 etc.389
 flexes of, 226
 Hilbert scheme, 140
 is a Castelnuovo curve, 188
 singularities, 44
 plane model, 76
 plane quartic curve, 266
 has 24 flexes, 227
 plane quintic, 121, 170, 220
 planetary motion, 361
 Plato, 354
 Poincaré sheaf, 97, 243
 point, defined, 8
 pointed curve, 250
 pole and polar relative to a conic, 357
 polynomial equation, numerical solution,
 355
 polynomial function, 83
 polynomial ring, 309
 Poncelet, Jean Victor, 358, 364
 porism, Poncelet, 358
 positive characteristic, 179, 207, 224, 237
 power series, 366
 Prabhakar Rao, A., 269
 preservation of number, 134
 primary decomposition, 8
 prime
 associated, 69
 characteristic, *see under* positive
 principal
 divisor, 13
 principal component, 337
 of Hilbert scheme, 348
 principal diameter, 355
 principal homogeneous space, 97
 principle of conservation of number, 242
 Pringsheim, Alfred, 370
 projective algebraic group, 97
 projective coordinates, 367
 projective dimension, 310
 projective geometry, 356, 358, 359, 362
 synthetic versus analytic, 359
 versus Euclidean, 359
 projective scheme, degree of, 52
 projective space, 8, 367
 bundle, 298
 complex, 367
 projectively homogeneous, 62, 225
 projectively normal, 69
 proper transform, 50
 Puiseux, Victor, 366, 371
 pullback of line bundle versus invertible
 sheaf, 17
 pure-dimensional, 69
 Pythagorean theorem, 354

 quadratic curve, 354, *see also* conic
 quadratic form, 125
 quadratic transformation, 371
 quadratically normal, 59, 69
 quadric surface
 classification of $-s$, 51
 containing the rational normal curve, 61
 containing three given lines, 285
 Gauss map, 171
 in \mathbb{P}^3 , 199
 in \mathbb{P}^4 , singular, 175
 pencil of $-s$, 86, 87
 quartic curve, 266, 364
 28 bitangents, 123
 elliptic, 87
 of genus 3, 121, 122
 rational, 64
 with a node or cusp, 118, 120
 quasi-Gorenstein, 316
 quasiprojective scheme, 8, 95, 108
 quasiprojective variety, 8, 84
 quaternions, 58
 quintic curve, 171, 177
 elliptic, 88, 219
 hypersurface, 175
 models, 121
 of genus 2, 119, 130
 plane, 328

- rational, 182
- quotient, 108
 - of schemes, 95
- $2r + 3$ lemma, 296
- ramification, 366
 - divisor, 34, 111, 275
 - index, 34, 288
 - point, 36, 111, 227
 - sequence, **224**, 232–238
 - simple, 252
- ramphoid, 172
- rank
 - of coherent sheaf, 16
 - of map of free modules, **314**
- Rao, A. Prabhakar, 269
- Rathmann, Jürgen, 180, 194
- rational
 - g -cuspidal curve, 246
 - curve, 73
 - smooth, 350
 - differential, 254
 - normal curve, 20, 21, 59–62, 71–73, 114, 195, 206, 224, 242
 - normal scroll, 289, 326
 - quartic, 64, 72, 338
- reduced divisor, 109
- reflexive, 273
- regular
 - differential, 77, 78, 254
 - function, 83
 - local ring, 69, 310
 - projection, 22
 - sequence, 68, 86
- residual divisor, 88
- residuation, 76
- residue, 40, 126
 - map, 274
- resolution of singularities, 55, *see also*
 - blowup
- restricted fiber power, **200**, 202
- restricted Hilbert scheme, 335
- restriction of ideals, 71
- resultant, 363
- ribbon, 287
- Riedl, Eric, 339
- Riemann, Georg Friedrich Bernhard, 11, 38, 76, 123, 368
- Riemann inequality, 368
- Riemann surface, 11, 98, 114
- Riemann–Hurwitz formula, 34, 85, 385
- Riemann–Roch formula, 102, 128, 227, 350, 379
- Riemann–Roch theorem, 57, 76, 83, 86, 90, 104, 106, 118, 119, 121, 167, 169, 173, 186, 209, 211, 227, 269, 343, 347, 368, 372, 381
 - easy, 30
 - for surfaces, 50
 - general, 284
 - generalization by Macaulay, 76
 - geometric, 45, 107, 169, 174, 175, 191, 193, 300, 327, 384
- right adjoint, 276
- rigid curve, 337, 345
- ring of invariants, 154
- Robson, Vicky, xv
- Roch, Gustav, 38, 368
- rope, 273, 286
- ruling, 51, 87, 169, 285, 290
 - interchange of $-s$, 199
- Salmon, George, 2, 129, 362
- Sammartano, Alessio, 292
- Samuel, Pierre, 180
- Satake compactification, 154
- saturated ideal, 142, 273, 286
- scheme
 - irreducible, 8
 - normal, 14
 - reduced, 8
- Schiaparelli, Giovanni Virginio, 371
- Schlessinger, Michael, 337
- Schubert class, 234, 251
- Schubert condition, 232
- Schubert cycle, special, 233, 234
- Schubert variety, 232
- scroll, 176, 289–307, *see also* rational
 - normal
- secant flag, 237
- secant plane conjecture, 68
- secant plane lemma, 107
- secant variety, 189, 206
- Segre embedding, 23, 51, 119, 145
- Seidel, Philipp, 370
- self-intersection, 109
- semicontinuity
 - of cohomology, 249
 - of degree, 181
 - of fiber dimension, 243
- semistable point, **155**
- semistable reduction, 152
- septic curve, 158

- series, *see* linear series
- Serre, Jean-Pierre, 310
- Serre duality, 37, 274, 279
- Serre's criterion, 69
- Serre–Grothendieck vanishing theorem, 29, 142, 379
- set-theoretic equality, 12, 59, 65, 87, 155, 182
- Severi, Francesco, 158, 162, 370
- Severi variety, 136, **162**, 205
- sextic curve, 90, 219
 - in \mathbb{P}^3 , 341
 - with 4 nodes, 217
 - with triple point, 219
- sheaf
 - canonical, *see under* canonical
 - coherent, 15
 - dualizing, 31
 - invertible, 15, 16, 18
 - of differentials, 31, 75
- sheet, **115**
 - number of –s, 198
- Shustin, Eugenii, 225
- ΣD , **83**
- Σ_d^r , **106**, **191**
- simple branch point, 116, 129
- simply branched cover, 159
- singular affine curve, 53
- singular scroll, 304
- singularity, 191, *see also* node, cusp, tacnode
 - higher, 105, 366, 369, 370
 - of planar curve, 44
- skew lines
 - 3 determine a quadric, 285
 - in \mathbb{P}^3 , 71
 - in \mathbb{P}^3 , 289
- skew-symmetric form, 73
- SL_n , $SL(V)$, representation theory of, 72
- small Severi variety, **162**
- smooth
 - curves, moduli of, 135
 - implies Cohen–Macaulay, 69
 - projective curve birational to an affine curve, 53
- smoothable, 335
- smoothing, 245
- Sottile, Frank, 225
- span of effective divisor, 45
- special divisor, 46
- special Schubert cycle, 233
- spectrum, 95
- split exact, 319
- square root of canonical sheaf, 123
- square-square, square-cube, 360
- stable, 152, **155**, 155, **156**
- stack, 157
- stacks of coins, 234
- stalk, 17
- stat, **180**
- stationary secant, 180, 190
- Staudt, Karl von, 359
- Steiner, Jakob, 358, 366, 371
- Steiner prize, 269
- Stillman, Mike, 313
- Stokes' theorem, 41
- Stolz, Otto, 366
- strange curve, **180**
- strict transform, 306
- strictly semistable point, **155**
- strong secant plane lemma, 191
- subschemes, family of, 137
- Sullivan, Arthur, 3
- sum of linear series, 23
- super-perfect ideal, 76
- superabundance, 38
- Sylvester, James Joseph, 363
- Sym, Sym^i , **8**, 59, 72, 73, 379, 380, 388
- symmetric function, 95
- symmetric group, 95, 115, 148, 198, 199, 203
- symmetric Koszul complex, 311
- symmetric power, 93, 96, 135
 - of a quasiprojective scheme, 95
- symmetric product, 101
- symmetric square, 108
- synthetic geometry, 359
- system (linear), *see* linear series
- syzygy, 32, 123
- syzygy matrix, 321
- syzygy theorem, *see under* Hilbert
- Szpiro, Lucien, 269
- tacnode, 44, 172, 264
 - triple, 266
- tangent, 355
- tangent space, 137
- tangential degeneracy, 206
- tautological invertible sheaf, 298
- tautological line bundle, 298
- tautological map, 318
- tautological subbundle, 143
- Theon of Smyrna, 354

- theta characteristic, 123–129
 number of $-s$, 125
- theta function, 123
- Thom–Porteous formula, 244
- Tor, 310
- torsion point, 101, 239
- torsion-free sheaf, 247
- torus generator, 72
- total ramification, 129
- total transform, 257
- trace element, 318, 323
- trace map, 263, 283
- transcendental extension, 57
- transcendentality of π , 370
- transposition, 202, 384
- tricanonical curves, 343
- tricanonical linear series, 153
- trigonal, 167, 219, 305, 307, 327
 canonical curve, 177
 curve of genus 5, 307
- trigonometry, 355
- triple point, 44, 55
 in space, 265
 on sextic, 219
 types of, 266
- triple tacnode, 266
- trisecant, 327
- Trott quartic, 124
- twisted cubic, 20, 73, 122, 140, 287, 335
 as intersection of 3 quadrics, 72
 linkage of, 270
- unibranch, 266
- uniform position, 297
 lemma, 199
- unirational, 158
- universal family, 94, 96, 134, 136, 137
 nonexistence, 157, 165
- universal (hyper)plane section, 199, 203
- universal property, 101
- universal singular point, 162
- universal subbundle, 143
- University of California, Berkeley, xv, 3
- unmixed ideal, 273, 372
- unmixed ring, 69
- unstable, 155, 165
- $V_{d,g}$, 162
- Vahlen, Karl Theodor, 182
- Van der Waerden, Bartel Leendert, 355
- vanishing
 flag, 232
- of global section at a point, 16
- sequence, 223, 224
- theorem (Serre–Grothendieck), 29, 379
- variety, convention on, 8
 of minimal degree, classification, 291
- vector bundle, 31, 143, 243, 312
- van de Ven, Antonius, xv
- Vermeulen, Alexius Maria, 227
- Veronese
 map, 20, 46, 59, 63, 118, 384
 equation of, 71
 is projectively homogeneous, 62
 third, 165
 surface, 20, 60, 300, 306, 328
 in \mathbb{P}^5 , 220
- vertex, 51
- very ample, 23, 103, 284, 298
 birationally, 205
- Viète, François, 360
- Vogt, Isabel, 214
- Voisin, Claire, 228, 330
- W_4^1 , 175, 176
- W_d^r , 105, 190, 211, 243
- weak del Pezzo, 218
- web, 21
- Weierstrass, Karl, 369
- Weierstrass \mathcal{P} -function, 99
- Weierstrass gap sequence, 228
- Weierstrass point, 226–230, 239, 252
 number of $-s$, 229
- Weierstrass semigroup, 227
- weight, 72, 224, 226
- Weil, André, 93, 102
- Weil divisor, 14, 52, 305
- Wiener, Christian, 369
- Yoneda’s lemma, 94, 108
- Zariski, Oscar, 225, 281
- Zariski tangent space, 105, 137, 138, 287
 dimension of, 342
 to the Hilbert scheme, 138
- Zariski topology, 8
- Zentralblatt für Mathematik*, 242
- zero-dimensional ring, 69
- Zusammenhang, 11