
Contents

Preface	xi
Introduction	xiii
Part 1. Foundations	
Chapter 1. Convex Analysis	3
§1.1. Convex sets	3
§1.2. Convex functions	13
§1.3. Differential properties of convex functions	24
§1.4. Problems	30
Chapter 2. Set-Valued Analysis	31
§2.1. Set-valued maps	31
§2.2. Derivatives of set-valued maps	37
§2.3. Lipschitzian approximations	41
§2.4. Extension theorem	44
§2.5. Fixed point theorems	47
§2.6. Convex processes	49
§2.7. Structure of a convex process	56
§2.8. Problems	61

Chapter 3. Nonsmooth Analysis	65
§3.1. Method of metric approximations	65
§3.2. Mordukhovich normal cone	67
§3.3. Separation theorem for nonconvex sets	72
§3.4. Nonsmooth calculus	75
§3.5. Lagrange multipliers	82
§3.6. Problems	83
Part 2. Differential Inclusions	
Chapter 4. Existence Theorems	87
§4.1. Background notes	88
§4.2. Lipschitzian differential inclusions	90
§4.3. Upper semi-continuous differential inclusions	96
§4.4. Discontinuous differential equations	103
§4.5. Existence of optimal solutions	106
§4.6. Dependence on initial conditions	109
§4.7. Discrete approximations	113
§4.8. Problems	116
Chapter 5. Viability and Invariance	119
§5.1. Monotone solutions to a differential inclusion	119
§5.2. Viability problem	122
§5.3. Invariant sets	127
§5.4. Existence of periodic solutions	130
§5.5. Pursuit in a differential game	132
§5.6. Problems	135
Chapter 6. Controllability	139
§6.1. Duality relation	139
§6.2. Controllability of convex processes	145
§6.3. Controllability at first approximation	147
§6.4. Controllability of some mechanical systems	152

§6.5. Problems	153
Chapter 7. Optimality	157
§7.1. Optimal solutions to discrete-time inclusions	157
§7.2. Optimal solutions to differential inclusions	160
§7.3. Time-optimal problem	165
§7.4. Problems	168
Chapter 8. Stability	171
§8.1. Lyapunov direct method	171
§8.2. Linear-selectionable differential inclusions	176
§8.3. Weak asymptotic stability of convex processes	185
§8.4. First approximation techniques	189
§8.5. Stability of a missile uniform motion	194
§8.6. Problems	196
Chapter 9. Stabilization	199
§9.1. Lyapunov functions for convex processes	200
§9.2. Stabilization problem	202
§9.3. Weak asymptotic stability and stabilizability	205
§9.4. Stabilizers for some mechanical systems	208
§9.5. Problems	210
Comments	213
Bibliography	219
Index	225