Preface

The notion of singularity is basic to mathematics. In elementary algebra singularity appears as a multiple root of a polynomial. In geometry a point in a space is non-singular if it has a tangent space whose dimension is the same as that of the space. Both notions of singularity can be detected through the vanishing of derivitives.

Over an algebraically closed field, a variety is non-singular at a point if there exists a tangent space at the point which has the same dimension as the variety. More generally, a variety is non-singular at a point if its local ring is a regular local ring. A fundamental problem is to remove a singularity by simple algebraic mappings. That is, can a given variety be desingularized by a proper, birational morphism from a non-singular variety? This is always possible in all dimensions, over fields of characteristic zero. We give a complete proof of this in Chapter 6.

We also treat positive characteristic, developing the basic tools needed for this study, and giving a proof of resolution of surface singularities in positive characteristic in Chapter 7.

In Section 2.5 we discuss important open problems, such as resolution of singularities in positive characteristic and local monomialization of morphisms.

Chapter 8 gives a classification of valuations in algebraic function fields of surfaces, and a modernization of Zariski's original proof of local uniformization for surfaces in characteristic zero.

This book has evolved out of lectures given at the University of Missouri and at the Chennai Mathematics Institute, in Chennai, (also known as Madras), India. It can be used as part of a one year introductory sequence in algebraic geometry, and would provide an exciting direction after the basic notions of schemes and sheaves have been covered. A core course on resolution is covered in Chapters 2 through 6. The major ideas of resolution have been introduced by the end of Section 6.2, and after reading this far, a student will find the resolution theorems of Section 6.8 quite believable, and have a good feel for what goes into their proofs.

Chapters 7 and 8 cover additional topics. These two chapters are independent, and can be chosen as possible followups to the basic material in the first 5 chapters. Chapter 7 gives a proof of resolution of singularities for surfaces in positive characteristic, and Chapter 8 gives a proof of local uniformization and resolution of singularities for algebraic surfaces. This chapter provides an introduction to valuation theory in algebraic geometry, and to the problem of local uniformization.

The appendix proves foundational results on the singular locus that we need. On a first reading, I recommend that the reader simply look up the statements as needed in reading the main body of the book. Versions of almost all of these statements are much easier over algebraically closed fields of characteristic zero, and most of the results can be found in this case in standard textbooks in algebraic geometry.

I assume that the reader has some familiarity with algebraic geometry and commutative algebra, such as can be obtained from an introductory course on these subjects. This material is covered in books such as Atiyah and MacDonald [**13**] or the basic sections of Eisenbud's book [**37**], and the first two chapters of Hartshorne's book on algebraic geometry [**47**], or Eisenbud and Harris's book on schemes [**38**].

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