
Contents

Preface	ix
Chapter 1. Money and Markets	1
Summary	1
§1.1. Introduction	1
§1.2. Money	2
§1.3. Interest Rates	3
§1.4. The Market	13
§1.5. Exercises	15
Chapter 2. Fair Games	17
Summary	17
§2.1. Fair Games	17
§2.2. Hedging and Arbitrage	21
§2.3. Exercises	26
Chapter 3. Set Theory	29
Summary	29
§3.1. Approaching Abstract Mathematics	29
§3.2. Infinity	33
§3.3. σ -Fields	40
§3.4. Partitions	48
§3.5. Filtrations and Information	52
§3.6. Exercises	55

Chapter 4. Measurable Functions	59
Summary	59
§4.1. Measurable Functions	59
§4.2. Convergence	69
§4.3. Exercises	74
Chapter 5. Probability Spaces	77
Summary	77
§5.1. Probability Spaces	77
§5.2. Call Options 1	83
§5.3. Independence	91
§5.4. Random Variables	100
§5.5. Stochastic Processes	103
§5.6. Exercises	104
Chapter 6. Expected Values	107
Summary	107
§6.1. Simple Random Variables	107
§6.2. Positive Bounded Random Variables	118
§6.3. Positive Random Variables	125
§6.4. Integrable Random Variables	133
§6.5. Summation of Series	139
§6.6. Exercises	142
Chapter 7. Continuity and Integrability	143
Summary	143
§7.1. Continuous Functions	143
§7.2. Convex Functions	146
§7.3. The Riemann Integral	151
§7.4. Independent Random Variables	156
§7.5. The Central Limit Theorem	161
§7.6. Exercises	163
Chapter 8. Conditional Expectation	165
Summary	165
§8.1. Call Options 2	165
§8.2. Conditional Expectation	171

§8.3. Hedging	182
§8.4. Exercises	186
Chapter 9. Lebesgue Measure	189
Summary	189
§9.1. Product Measures	189
§9.2. Lebesgue Measure	197
§9.3. Density Functions	203
§9.4. Exercises	208
Chapter 10. Martingales	209
Summary	209
§10.1. Discrete-Time Martingales	209
§10.2. Martingale Convergence	214
§10.3. Continuous-Time Martingales	220
§10.4. Exercises	225
Chapter 11. The Black-Scholes Formula	227
Summary	227
§11.1. Share Prices as Random Variables	227
§11.2. Call Options 3	233
§11.3. Change of Measure	239
§11.4. Exercises	242
Chapter 12. Stochastic Integration	243
Summary	243
§12.1. Riemann Sums	243
§12.2. Convergence of Random Variables	245
§12.3. The Stochastic Riemann Integral	251
§12.4. The Itô Integral	257
§12.5. Itô's Lemma	265
§12.6. Call Options 4	274
§12.7. Epilogue	277
§12.8. Exercises	279
Solutions	281
Bibliography	299
Index	301