

---

# Preface

*Say what you know, do what you must, come what may.*

–Sofya Kovalevskaya

*To us probability is the very guide of life.*

–Bishop Joseph Butler

A few years ago the University of Utah switched from the quarter system to the semester system. This change gave the faculty a chance to re-evaluate their course offerings. As part of this re-evaluation process we decided to replace the usual year-long graduate course in probability theory with one that was a semester long. There was good reason to do so. The role of probability in mathematics, science, and engineering was, and still is, on the rise. There is increasing demand for a graduate course in probability. And yet the typical graduate student is not able to tackle a large number of year-long courses outside his or her own research area. Thus, we were presented with a non-trivial challenge: Can we offer a course that addresses the needs of our own, as well as other, graduate students, all within the temporal confines of one semester? I believe that the answer to the preceding question is “yes.”

This book presents a cohesive graduate course in measure-theoretic probability that specifically has the one-semester student in mind. There is, in fact, ample material to cover an ordinary year-long course at a more leisurely pace. See, for example, the many sections that are entitled complements, and those that are called applications. However, the primary goals of this book are to maintain brevity and conciseness, and to introduce probability quickly and at a modestly deep level. I have used as my model a standard one-semester *undergraduate* course in probability. In that setting, the

instructional issues are well understood, and most experts agree on what should be taught.

Giving a one-semester introduction to graduate probability necessarily involves making concessions. Mine form the contents of this book: No mention is made of Kolmogorov's theory of random series; Lévy's continuity theorem of characteristic functions is sadly omitted; Markov chains are not treated at all; and the construction of Brownian motion is Fourier-analytic rather than "probabilistic."

That is not to say that there is little coverage of the theory of stochastic processes. For example, included you will find an introduction to Itô's stochastic calculus and its connections to elliptic partial differential equations. This topic may seem ambitious, and it probably is for some readers. However, my experience in teaching this material has been that the reader who knows some measure theory can cover the book up to and including the last chapter in a single semester. Those who wish to learn measure theory from this book would probably aim to cover less stochastic processes.

**Teaching Recommendations.** In my own lectures I often begin with Chapter 2 and prove the De Moivre–Laplace central limit theorem in detail. Then, I spend two or three weeks going over basic results in analysis [Chapters 3 through 5]. Only a handful of the said results are actually proved. Without exception, one of them is Carathéodory's monotone class theorem (p. 30). The fundamental notion of independence is introduced, and a number of important examples are worked out. Among them are the weak and the strong laws of large numbers [Chapter 6], respectively due to A. Ya. Khintchine and A. N. Kolmogorov. Next follow elements of harmonic analysis and the central limit theorem [Chapter 7]. A majority of the subsequent lectures concern J. L. Doob's theory of martingales (1940) and its various applications [Chapter 8]. After martingales, there may be enough time left to introduce Brownian motion [Chapter 9], construct stochastic integrals, and deduce a striking computation, due to Chung (1947), of the distribution of the exit time from  $[-1, 1]$  of Brownian motion (p. 197). If at all possible, the latter topic should not be missed.

My personal teaching philosophy is to showcase the big ideas of probability by deriving very few, but central, theorems. François Marie Arouet [Voltaire] once wrote that "the art of being a bore is to tell everything." Viewed in this light, a chief aim of this book is to not bore.

I would like to leave the reader with one piece of advice on how to best use this book. Read it thoughtfully, and with pen and paper.

**Acknowledgements:** This book is based on the combined contents of several of my previous graduate courses in probability theory. Many of these were given at the University of Utah during the past decade or so. Also, I have used parts of some lectures that I gave during the formative stages of my career at MIT and the University of Washington. I wish to thank all three institutions for their hospitality and support, and the National Science Foundation, the National Security Agency, and the North-Atlantic Treaty Organization for their financial support of my research over the years.

All scholars know about the merits of library research. Nevertheless, the role of this lore is underplayed in some academic texts. I, for one, found the following to be enlightening: Billingsley (1995), Breiman (1992), Chow and Teicher (1997), Chung (1974), Cramér (1936), Dudley (2002), Durrett (1996), Fristedt and Gray (1997), Gnedenko (1967), Karlin and Taylor (1975, 1981), Kolmogorov (1933, 1950), Krickeberg (1963, 1965), Lange (2003), Pollard (2002), Resnick (1999), Stroock (1993), Varadhan (2001), Williams (1991), and Woodroffe (1975). Without doubt, there are other excellent references. The student is encouraged to consult other resources in addition to the present text. He or she would do well to remember that it may be nice to know facts, but it is vitally important to have a perspective.

I am grateful to the following for their various contributions to the development of this book: Nelson Beebe, Robert Brooks, Pieter Bowman, Rex Butler, Edward Dunne, Stewart Ethier, Victor Gabrenas, Frank Gao, Ana Meda Guardiola, Jan Hannig, Henryk Hecht, Lajos Horváth, Zsuzsanna Horváth, Adam Keenan, Karim Khader, Remigijus Leipus, An Le, David Levin, Michael Purcell, Pejman Mahboubi, Pedro Méndez, Jim Pitman, Natalya Pluzhnikov, Matthew Reimherr, Shang-Yuan Shiu, Josef Steinebach, James Turner, John Walsh, Jun Zhang, and Liang Zhang. Many of these people have helped find typographical errors, and even a few serious mistakes. All errors that remain are of course mine.

My family has been a stalwart pillar of patience. Their kindness and love were indispensable in completing this project. I thank them deeply.

And last but certainly not the least, my eternal gratitude is extended to my teachers, past and present, for introducing me to the joys of mathematics. I hope only that some of their ingenuity and spirit persists throughout these pages.

Davar Khoshnevisan  
Salt Lake City, January 2007