
Contents

Preface	xi
General Notation	xv
Chapter 1. Classical Probability	1
§1. Discrete Probability	1
§2. Conditional Probability	4
§3. Independence	6
§4. Discrete Distributions	6
§5. Absolutely Continuous Distributions	10
§6. Expectation and Variance	12
Problems	13
Notes	15
Chapter 2. Bernoulli Trials	17
§1. The Classical Theorems	18
Problems	21
Notes	22
Chapter 3. Measure Theory	23
§1. Measure Spaces	23
§2. Lebesgue Measure	25
§3. Completion	28
§4. Proof of Carathéodory's Theorem	30
Problems	33

Notes	34
Chapter 4. Integration	35
§1. Measurable Functions	35
§2. The Abstract Integral	37
§3. L^p -Spaces	39
§4. Modes of Convergence	43
§5. Limit Theorems	45
§6. The Radon–Nikodým Theorem	47
Problems	49
Notes	52
Chapter 5. Product Spaces	53
§1. Finite Products	53
§2. Infinite Products	58
§3. Complement: Proof of Kolmogorov’s Extension Theorem	60
Problems	62
Notes	64
Chapter 6. Independence	65
§1. Random Variables and Distributions	65
§2. Independent Random Variables	67
§3. An Instructive Example	71
§4. Khintchine’s Weak Law of Large Numbers	71
§5. Kolmogorov’s Strong Law of Large Numbers	73
§6. Applications	77
Problems	84
Notes	89
Chapter 7. The Central Limit Theorem	91
§1. Weak Convergence	91
§2. Weak Convergence and Compact-Support Functions	94
§3. Harmonic Analysis in Dimension One	96
§4. The Plancherel Theorem	97
§5. The 1-D Central Limit Theorem	100
§6. Complements to the CLT	101
Problems	111
Notes	117

Chapter 8. Martingales	119
§1. Conditional Expectations	119
§2. Filtrations and Semi-Martingales	126
§3. Stopping Times and Optional Stopping	129
§4. Applications to Random Walks	131
§5. Inequalities and Convergence	134
§6. Further Applications	136
Problems	151
Notes	157
Chapter 9. Brownian Motion	159
§1. Gaussian Processes	160
§2. Wiener's Construction: Brownian Motion on $[0, 1)$	165
§3. Nowhere-Differentiability	168
§4. The Brownian Filtration and Stopping Times	170
§5. The Strong Markov Property	173
§6. The Reflection Principle	175
Problems	176
Notes	180
Chapter 10. Terminus: Stochastic Integration	181
§1. The Indefinite Itô Integral	181
§2. Continuous Martingales in $L^2(\mathbb{P})$	187
§3. The Definite Itô Integral	189
§4. Quadratic Variation	192
§5. Itô's Formula and Two Applications	193
Problems	199
Notes	201
Appendix	203
§1. Hilbert Spaces	203
§2. Fourier Series	205
Bibliography	209
Index	217