Introduction

The study of singularities of analytic functions can be considered as a subarea of the theory of functions of several complex variables and of algebraic/analytic geometry. It has in the meantime, together with the theory of singularities of differentiable mappings, developed into an independent subject, singularity theory. Through its connections with very many other mathematical areas and applications to natural and economic sciences and in technology (for example, under the heading 'catastrophe theory') this theory has aroused great interest. The particular appeal, but also its particular difficulty, lies in the fact that deep results and methods from various branches of mathematics come into play here.

The aim of this book is to present the foundations of the theory of functions of several complex variables and on this basis to develop the fundamental concepts of the theory of isolated singularities of holomorphic functions systematically. It is derived from lectures given by the author to mathematics students in their third and fourth year to introduce them to current research questions in the area of the theory of functions of several variables. The book has its genesis in this. As prerequisites we assume only an introductory knowledge of the theory of functions of a single complex variable and of algebra, such as students will normally acquire in their first two years of study. The first two chapters correspond to a continuation of the course on complex analysis and deal with Riemann surfaces and the theory of functions of several complex variables. They also present an introduction to local complex geometry. In the third chapter the results will be applied to deformation and classification of isolated singularities of holomorphic functions. These three chapters have grown from notes for the author's lectures on Riemann surfaces and the theory of functions of several complex variables delivered in Hanover in the winter semester of 1998/1999 and the summer semester of 1999. Parts of these notes go back to similar courses given in the winter semester of 1992/1993 and the summer semester of 1993.

The rest of the book deals with the topological study of these singularities begun in the now classical book of J. Milnor [**Mil68**]. Picard-Lefschetz theory is an important tool and can be viewed as a complex version of Morse theory. It is expounded at the beginning of the second volume of the extensive two-volume standard work of V. I. Arnold, S. M. Gusein-Zade and A. N. Varchenko [**AGV85, AGV88**]. These books assume considerable prior knowledge. We offer an introduction to this theory in the last two chapters of the present book. In the fourth chapter we first present the necessary foundations of algebraic and of differential topology. The fifth chapter introduces the topological study of singularities. It rests in part on [**AGV88**, Part I. The topological structure of isolated critical points of functions]. At the end of this chapter there is a survey of topical results, some presented without proof. The last two chapters are based on a course on singularities delivered by the author in Hanover in the winter semester of 1993/1994.

This book can be used for a course on functions of several complex variables, an introductory course on differential topology, or for a special course or seminar on an introduction to singularity theory. The first two chapters would be suitable for a further course on functions of several complex variables. The beginning of §1.1, §1.2, and the first four sections of Chapter 3 and Chapter 4 treat themes from differential topology and can be read independently of the rest of the book: they can therefore serve as the basis of an introductory course on differential topology. Chapter 3 and Chapter 5 can be used as reading for a seminar on *Introductory singularity theory*, with reference back to the results of the previous chapters according to the state of knowledge of the participants.

Naturally the themes discussed here are only a small choice from a great variety of possibilities. This choice has been shaped by the author's own predilections and by his work. Nevertheless the author hopes that his book presents a good foundation for the study of the more advanced literature indicated in the bibliography.

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