Preface

This book is based on graduate courses taught by the author over the last fourteen years in the mathematics department of Stony Brook University. The goal of these courses was to introduce second year graduate students with no prior knowledge of physics to the basic concepts and methods of quantum mechanics. For the last 50 years quantum physics has been a driving force behind many dramatic achievements in mathematics, similar to the role played by classical physics in the seventeenth to nineteenth centuries. Classical physics, especially classical mechanics, was an integral part of mathematical education up to the early twentieth century, with lecture courses given by Hilbert and Poincaré. Surprisingly, quantum physics, especially quantum mechanics, with its intrinsic beauty and connections with many branches of mathematics, has never been a part of a graduate mathematics curriculum. This course was developed to partially fill this gap and to make quantum mechanics accessible to graduate students and research mathematicians.

L.D. Faddeev was the first to develop a course in quantum mechanics for undergraduate students specializing in mathematics. From 1968 to 1973 he regularly lectured in the mathematics department of St. Petersburg State University in St. Petersburg, Russia[1], and the author enjoyed the opportunity to take his course. The notes for this book emerged from an attempt to create a similar course for graduate students, which uses more sophisticated mathematics and covers a larger variety of topics, including the Feynman path integral approach to quantum mechanics.

[1]At that time in Leningrad, Soviet Union.

There are many excellent physics textbooks on quantum mechanics, starting with the classic texts by P.A.M. Dirac [**Dir47**], L.D. Landau and E.M. Lifshitz [**LL58**], and V.A. Fock [**Foc78**], to the encyclopedic treatise by A. Messiah [**Mes99**], the recent popular textbook by J.J. Sakurai [**Sak94**], and many others. From a mathematics perspective, there are classic monographs by J. von Neumann [**vN96**] and by H. Weyl [**Wey50**], as well as a more recent book by G.W. Mackey [**Mac04**], which deal with the basic mathematical formalism and logical foundations of the theory. There is also a monumental project [**DEF$^+$99**], created with the purpose of introducing graduate students and research mathematicians to the realm of quantum fields and strings, both from a mathematics and a physics perspective. Though it contains a very detailed exposition of classical mechanics, classical field theory, and supersymmetry, oriented at the mathematical audience, quantum mechanics is discussed only briefly (with the exception of L.D. Faddeev's elegant introduction to quantum mechanics in [**Fad99**]). Excellent lecture notes for undergraduate students by L.D. Faddeev and O.A. Yakubovskiĭ [**FY80**] seems to be the only book on quantum mechanics completely accessible to mathematicians[2]. Recent books by S.J. Gustafson and I.M. Sigal [**GS03**] and by F. Strocchi [**Str05**] are also oriented at mathematicians. The latter is a short introductory course, while the former is more an intermediate level monograph on quantum theory rather than a textbook on quantum mechanics. There are also many specialized books on different parts of quantum mechanics, like scattering theory, the Schrödinger operator, $\mathbb{C}^*$-algebras and foundations, etc.

The present book gives a comprehensive treatment of quantum mechanics from a mathematics perspective and covers such topics as mathematical foundations, quantization, the Schrödinger equation, the Feynman path integral and functional methods, and supersymmetry. It can be used as a one-year graduate course, or as two one-semester courses: the introductory course based on the material in Part 1, and a more advanced course based on Part 2. Part 1 of the book, which consists of Chapters 1-4, can be considered as an expanded version of [**FY80**]. It uses more advanced mathematics than [**FY80**], and contains rigorous proofs of all main results, including the celebrated Stone-von Neumann theorem. It should be accessible to a second-year graduate student. As in [**FY80**], we adopt the approach, which goes back to Dirac and was further developed by Faddeev, that classical mechanics and quantum mechanics are just two different realizations of the fundamental mathematical structure of a physical theory that uses the notions of observables, states, measurement, and the time evolution — dynamics. Part 2, which consists of Chapters 5-8, deals with functional methods in quantum

[2]The English translation will appear in the AMS "The Student Mathematical Library" series.

mechanics, and goes beyond the material in [**FY80**]. Exposition there is less detailed and requires certain mathematical sophistication.

Though our presentation freely uses all the necessary tools of modern mathematics, it follows the spirit and tradition of the classical texts and monographs mentioned above. In this sense it can be considered "neoclassical" (as compared with a more abstract approach in [**DF99a**]). Each chapter in the book concludes with a special *Notes and references* section, which provides references to the necessary mathematics background and physics sources. A courageous reader can actually learn the relevant mathematics by studying the main text and consulting these references, and with enough sophistication, could "translate" corresponding parts in physics textbooks into the mathematics language. For the physics students, the book presents an opportunity to become familiar with the mathematical foundations and methods of quantum mechanics on a "case by case" basis. It is worth mentioning that development of many mathematics disciplines has been stimulated by quantum mechanics.

There are several ways to study the material in this book. A casual reader can study the main text in a cursory manner, and ignore numerous remarks and problems, located at the end of the sections. This would be sufficient to obtain basic minimal knowledge of quantum mechanics. A determined reader is supposed to fill in the details of the computations in the main text (a pencil and paper are required), which is the only way to master the material, and to attempt to solve the basic problems[3]. Finally, a truly devoted reader should try to solve all the problems (probably consulting the corresponding references at the end of each section) and to follow up on the remarks, which may often be linked to other topics not covered in the main text.

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[3] We leave it to the reader to decide which problems are basic and which are advanced.