

Introduction

THE THIRTEENTH EDITION of *What's Happening in the Mathematical Sciences* reminds us that “what’s new” is often very closely related to “what’s old.” Good, substantive problems and techniques often continue to deliver fresh results for many years and multiple generations—just as a tree with a good rootstock continues to bear fruit for years.

A beautiful example is the resuscitation of a century-old proof technique in number theory called the “circle method” to prove that there are infinitely many primes missing the digit “7”—or any other single digit you might choose. (“Missing One Digit,” p. 86) In “One Stone to Rule Them All” (p. 2), we describe how a self-described “shape hobbyist” discovered a new shape (yes, just for Volume 13, a 13-sided shape) that almost overnight turned out to be a holy grail that tiling theorists had been seeking for half a century. “Organizing the Chaos Inside the Brain” (p. 34) explores how neuroscientists are using strange attractors (a mathematical concept discovered more than 60 years ago) to understand the causal relationship between neurons: or perhaps more poetically, to “read the mind” of a fruit fly. And in “How to Draw an Alternate Universe” (p. 18) geometers use computers and virtual reality to bring to life the eight distinct geometries of three-dimensional “alternate universes,” discovered more than 40 years ago by Bill Thurston. The results are truly mind-bending.

Sometimes ideas in one branch of mathematics turn another branch completely upside down. A case in point is “A Fascination of Spheres” (p. 114), where mathematicians found an amazing proof that the densest packing of spheres in 8 and 24 dimensions are lattice packings. The solution involves the construction of “magic functions” that most mathematicians (if they had any opinion at all) would probably have said could not exist. In “How Mathematicians Unearthed the Stubborn Secrets of Fano Varieties” (p. 58), algebraic geometers seeking a classification of “Fano varieties” (one of the building blocks of surfaces defined by polynomial equations) were forced to import a decidedly non-algebraic concept known as “K-stability.”

Real-world problems are always a fertile source of mathematics. In “Multi-View Geometry: *E Pluribus Unum*” (p. 46) the seemingly simple problem of reconstructing a 3-dimensional scene from 2-dimensional photographs leads us down some unexpected byways, some of them almost as old as photography itself. And we find out where Abe Lincoln gave his Gettysburg Address! (No, that’s not a trick question.) In “Fluid Flow: Two Paths to a Singularity” (p. 98), two groups hope to be the first to determine whether Eulerian fluids (i.e., fluids with zero viscosity) can develop singularities. This would be somewhat like the discovery of black holes: using the equations of physics to predict the existence of points where the equations themselves break down.

And finally, A.I. Machine learning exploded into the public consciousness in 2022, with the introduction of ChatGPT. Artificial intelligence in the past has been a field with as many myths as successes, but mathematicians (“Deep Learning: Part Math, Part Alchemy,” p. 70) play an important role in sorting out which abilities and which liabilities of the latest generation of neural networks are genuine.

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