

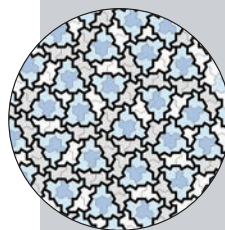
# Contents

---

## 2 **One Stone to Rule Them All**

Dana Mackenzie

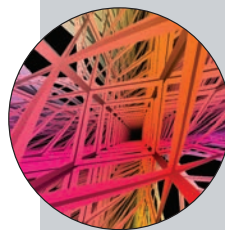
It was like a jigsaw puzzle where you not only have to fit the pieces together, but you have to design them yourself! In 2022, a group of four mathematicians discovered the first known *ein stein*: A single shape that is capable of tiling the Euclidean plane, but does not admit any periodic tilings. The thirteen-sided puzzle piece was nicknamed “the hat,” because it vaguely resembles a Panama hat. However, that was not the only surprise: it turned out to be one in an infinite family of pieces with a similar “aperiodicity” property, and it challenged the conventional wisdom about what is a quasicrystal.



## 18 **How to Draw an Alternate Universe**

Leila Sloman

When you flip the light switch in your living room, rays of light zip from the bulb to objects in the room and bounce back into your eye. This creates a picture of the room, helping you to avoid tripping on your furniture. But if the geometry of our universe was very different—changing the way shapes and lines fit together, and thus the way light navigates through space—the picture of your living room would also look different. The mathematician Bill Thurston predicted that there are eight basic possibilities for the geometry of a 3D space. Over the past several years, multiple groups have succeeded in creating images of all eight.



## 34 **Organizing the Chaos Inside the Brain**

Leila Sloman

What causes a fruit fly to walk around on a Styrofoam ball? A good answer, perhaps, is “its brain!” In research from 2021, a team identified causal links among groups of neurons in a fly brain. With their findings, they produced simulations of fly movement. Their strategy goes back to a 1981 theorem from chaos theory that made its way from physics to ecology and has expanded to biology, climate science, and more.



## 46 **Multi-view Geometry: *E Pluribus Unum***

Dana Mackenzie

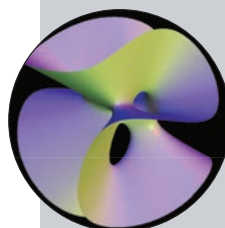
In 2022, a former Disney animator used computer software to give a probably definitive answer to a question that has long puzzled Civil War buffs: Where exactly was Abraham Lincoln standing when he gave his famous Gettysburg address? Reconstructing a three-dimensional scene from a small number of two-dimensional photos is a fascinating challenge, involving mathematical techniques that go back to the time of Lincoln, and even earlier.

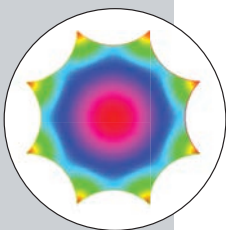
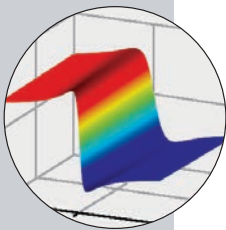
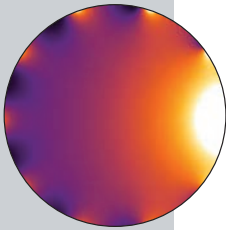
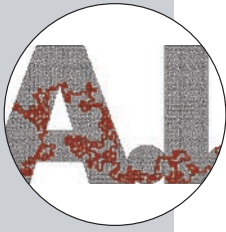


## 58 **How Mathematicians Unearthed the Secrets of Fano Varieties**

Leila Sloman

Mathematicians dealing with classes of complicated objects often want to find an organized arrangement of those objects. But sometimes that arrangement, called a moduli space, can have funny and unwanted features. For years, these features popped up when mathematicians tried to find a moduli space for objects known as “Fano varieties,” a certain type of solution set for polynomial equations. A line of research culminating in 2022 used a concept from differential geometry called “K-stability” to identify problematic Fano varieties, making the moduli space perfect.





---

## 70 **Deep Learning: Part Math, Part Alchemy**

Dana Mackenzie

One of the biggest science stories of the year in 2022 was the seemingly abrupt emergence of artificial intelligence, or “AI.” While many of the abilities and liabilities of systems like ChatGPT and GPT-4 remain very unclear, the systems are based on simple mathematical concepts. Several mathematicians have taken on the challenge of separating hype from reality, and explaining (for example) why certain neural networks are prone to making mistakes, how to reduce the mistakes, and how to identify documents written by AI through digital watermarking. These problems are a fascinating mix of mathematics, computer science, and statistics.

## 86 **Missing One Digit**

Dana Mackenzie

Can you find a prime number with no 7’s in it? Of course you can: 3, 5, 11, 13, 651665054896013834541, ... But until recently, nobody could say whether this list goes on forever. Finally, an Oxford number theorist managed to prove that it does—and he used a century-old method (the “circle method”) that was invented at Cambridge.

## 98 **Fluid Flows: Two Paths to a Singularity**

Dana Mackenzie

A million-dollar unsolved problem in fluid mechanics asks whether an initially smoothly flowing fluid can develop a “singularity”—a point where smooth motion cannot be continued and the flow becomes unpredictable. While the full problem remains unsolved, two groups of mathematicians are closing in on an answer in the case of fluids with zero viscosity. One group has shown that singularities can occur at a boundary, such as a pipe wall. Both groups use the computer (and one uses machine learning) to identify potentially singular structures.

## 114 **A Fascination of Spheres**

Dana Mackenzie

The densest possible packing of disks on a plane has probably been known since prehistory: a honeycomb arrangement. In three dimensions, the densest packing has also been intuitively known for centuries, but only proven to be densest in 1998. Given this slow progress, you might think that the densest possible packing of spheres in higher-dimensional space would remain forever unknown. But one mathematician made a breakthrough in 2016, which rapidly led to the proof of the densest sphere arrangements in 8 and 24 dimensions. Her discovery relates sphere packings to quantum field theories in physics, as well as other unsolved problems in geometry.