Contents

1 Background 1
- 1.1 Axioms of set theory 1
- 1.2 Constructing numbers from sets 6
- 1.3 Set theoretic construction of the real numbers 10
- 1.4 Sequences of real numbers 18
- 1.5 Infinite series 23
- 1.6 Sequences of functions 29
- 1.7 Power series 35
- 1.8 Metric spaces and Euclidean spaces 40
- 1.9 The Heine-Borel theorem 48
- 1.10 Vector-valued functions 50
- 1.11 Derivatives of multivariable functions 52
- 1.12 The inverse function theorem 62
- 1.13 The implicit function theorem 67
- 1.14 The Lagrange multiplier method 71
- 1.15 Level sets and tangent spaces 73
- 1.16 Changing variables in integrals 75
- 1.17 Volume and surface area of the hypersphere 80
- 1.18 Green's theorem 82
- 1.19 Theorems of Gauss and Stokes 87
- 1.20 Differential forms 91

2 Measure theory 101
- 2.1 Topological spaces and measure spaces 101
- 2.2 The Lebesgue integral 104
- 2.3 Inner product spaces 110
- 2.4 Orthonormal sets 116
- 2.5 Trigonometric series 121
- 2.6 Banach spaces 125
- 2.7 Baire's theorem 131
- 2.8 Hahn-Banach theorem 133

2.9 Examples of dual spaces . . . 137

3 Fourier Transforms . . . 143
3.1 Fubini's theorem and convolutions . . . 143
3.2 The Fourier Transform . . . 146
3.3 Differentiation under the integral sign . . . 150
3.4 Further examples of Fourier transforms . . . 154
3.5 A convolution theorem . . . 156
3.6 The inversion theorem . . . 157
3.7 Further properties of the Fourier transform . . . 162
3.8 The Plancherel theorem . . . 163
3.9 The Uncertainty Principle . . . 166
3.10 Trigonometric polynomials . . . 170
3.11 The isoperimetric inequality . . . 173
3.12 Weyl's criterion and uniform distribution . . . 176
3.13 Fourier series . . . 179
3.14 The Poisson summation formula . . . 182
3.15 A Fourier analytic proof of the central limit theorem . . . 189

4 Complex Analysis . . . 193
4.1 Basic definitions . . . 193
4.2 Integration over paths . . . 200
4.3 The local Cauchy theorem . . . 203
4.4 Zeros and singularities . . . 209
4.5 The maximum modulus principle . . . 214
4.6 The Global Cauchy Theorem . . . 221
4.7 The Calculus of Residues . . . 224
4.8 Further examples . . . 229
4.9 Rouché's Theorem . . . 236
4.10 Infinite Products and Weierstrass Factorization . . . 240
4.11 The logarithm . . . 247
4.12 The Phragmén-Lindelöf Theorem and Jensen's Theorem . . . 252
4.13 Entire Functions of Order 1 . . . 257
4.14 The Gamma Function . . . 260
4.15 Stirling's formula . . . 264
4.16 The Wiener-Ikehara Tauberian theorem . . . 269
4.17 The analytic theorem . . . 271
4.18 The proof of the Tauberian theorem . . . 274
4.19 The prime number theorem . . . 277
4.20 Further applications . . . 280
4.21 The Paley-Wiener theorems . . . 282

5 Introduction to Algebraic Topology 289
5.1 A very brief historical introduction 289
5.2 Homotopic paths 291
5.3 The Fundamental Group 296
5.4 Examples of some fundamental groups. 297
5.5 Covering spaces 302
5.6 Applications 307
5.7 Group actions and orbit spaces 312
5.8 Automorphisms of covering spaces 317
5.9 The universal covering space 322
5.10 Suggestions for further reading 326
References 329

Index 331