Introduction

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Emil Artin was one of my intellectual heroes. As an undergraduate math major I studied, in detail, his well-known booklet *Galois Theory* and spent a summer mastering his wonderful book *Geometric Algebra*. Moreover, in class I learned algebra from the famous textbook *Modern Algebra* of B. L. van der Waerden, which was based, at least in part, on lectures of Emil Artin and Emmy Noether. Artin's influence seemed to be everywhere, and not just in these expository works. He was a renowned research mathematician, one of the most important algebraists and number theorists of the twentieth century. Already during this early stage of my mathematical education, I had heard of the Artin reciprocity law, Artin L-functions, Artinian rings, and the famous Artin conjectures. For all these reasons, I formed the ambition of attending Princeton University as a graduate student and studying under his direction. Unfortunately, when I arrived at Princeton in the fall of 1959 he was no longer on the faculty. He had accepted a position at Hamburg University in Germany.

Luckily, even without the author, the printed word remained. I continued to learn from Artin's expository writings. His works have a gem-like quality, brilliant in form and content. Unfortunately, many of them are no longer readily available. This small volume represents a modest attempt to correct this situation. The reader will find assembled here a selection of Artin's books and articles, all of which retain their power to both instruct and delight.

Part 1

Emil Artin was born in Vienna on March 3, 1898. His father was an art dealer and his mother an opera singer. Art and music were important to him throughout his life. When he was only four years old, his father died and he was sent to live with his grandmother. Two years later his mother remarried and he returned to live with her and his new stepfather, who was the owner of a cloth factory in Reichenberg, Bohemia. With the exception of a happy year of study in Paris at the age of fourteen, Artin lived in Reichenberg until his graduation from high school. After only one semester at the University of Vienna, he was drafted into the Austrian army where he served until the end of the First World War.

In January of 1919 he entered the University of Leipzig where he studied mathematics under the supervision of Gustav Herglotz. His relationship with Herglotz was excellent. According to Serge Lang and John Tate, his future Ph.D. students, Herglotz was the only person that Artin recognized as his "teacher". He made rapid strides. Within two years he was awarded a doctorate on the basis of an outstanding thesis, a seminal work that influenced many later developments in number theory and algebraic geometry. After graduating from the University of Leipzig, Artin went to Göttingen, the center of the mathematical world at that time. Although he stayed there only a year, this period turned out to be very important for his future development, in particular because it was there that he became aware of the great paper of Teiji Takagi on class field theory. In an attempt to better understand Takagi's results Artin was led to formulate what is now known as the Artin reciprocity law, undoubtedly his crowning mathematical achievement, a pinnacle of algebraic number theory. The search for a proof and the successful completion of that search did not take place at Göttingen. Artin moved in 1922 to the relatively new University of Hamburg, which soon became a great center of mathematical activity. It was here that he discovered the proof of his reciprocity law. Artin moved up rapidly through the academic ranks, becoming a Full Professor in 1926 at the age of 28. Thereafter he, together with Erich Hecke and Wilhelm Blaschke, directed the activities of the Mathematical Seminar of Hamburg. To get some idea of the rich mathematical life of that time and place, one only has to leaf through issues of the *Abhandlungen der Mathematisches Seminar Hamburg* of that period.

The years 1921–1931 were amazingly productive for Artin. As Richard Brauer expresses it in his valuable article on Artin's career [20], "The ten year period 1921–1931 of Artin's life had seen an activity not often equalled in the life of a mathematician". In his thesis Artin developed the theory of hyperelliptic curves over finite fields on the model of the theory of quadratic number fields. In so doing he introduced new types of zeta and L-functions and proved in special cases a new type of Riemann hypothesis connected with these functions. He discovered and proved the Artin reciprocity law (generalizing in one blow all previous reciprocity laws). He invented Artin L-functions and proved many of their most important properties. Moreover, he made a conjecture about them that remains to this day one of the great unsolved problems of number theory. All this work was in number theory. In topology he invented the braid group and proved a series of important results about them. Braid groups turn out to be of great importance in topology, algebraic geometry, and, in recent years, physics. Within algebra he invented the abstract theory of real fields and was led to a solution of Hilbert's problem number 17. He also contributed important papers to the theory of hypercomplex systems which are now called associative algebras. He helped extend and generalize earlier work of Leonard Dickson and Joseph H. M. Wedderburn. Even this list does not exhaust the many important contributions he made during this immensely productive period.

In 1929, Artin married Natasha Jasny. She had been one of his students. After their marriage, she continued to follow developments in mathematics and was later an editor of the journal *Communications on Pure and Applied Mathematics*. The young couple had two children, Michael and Karin, while living in Germany, and a third child, Tom, after they moved to the United States. Michael developed into a very important mathematician in his own right. Unfortunately for the Artin family, the early years of their marriage were clouded by the worldwide Depression and the rise to power of the Nazi Party. According to Brauer, after 1933 "It was only a question of time before Artin, with his feeling for individual freedom, his sense of justice, his abhorrence of physical violence, would leave Germany". In 1937, Artin and his family left Germany and emigrated to the United States.

He spent a year at Notre Dame in South Bend, Indiana, and then moved to Indiana University at Bloomington. He continued to give lectures at Notre Dame, and it is there that he published his famous book on Galois theory which we reproduce in this volume. He and his family spent eight happy years in Bloomington. During this time he entered into a productive collaboration with George Whaples, which resulted in several influential joint papers. Two of these, on the product formula, are reproduced in this volume. Another paper concerns rings with the descending chain condition on ideals. Such rings are now called Artinian rings. A small book on this subject, *Rings with Minimal Condition*, was written by Artin during this period [8] (co-authored with C. J. Nesbitt and R. M. Thrall). Bloomington was congenial both professionally and socially. However, when Artin received an offer to join the faculty of Princeton University in 1946, he took it. The high quality of the Princeton Mathematics Department, the proximity of the Institute for Advanced Study, and the prospect of first-rate graduate students must have been lures too difficult to resist. At Princeton, he became interested once again in the foundations of class field theory. A new approach was afforded by the use of group cohomology, which was introduced into the subject by Gerhardt Hochschild, Tadashi Nakayama, and André Weil. This approach was developed and extended further in Artin's seminar at Princeton. In the seminar were two of Artin's most illustrious graduate students, Serge Lang and John Tate. Tate made important contributions to class field theory, which led to significant generalizations of Artin's work on reciprocity. The Artin–Tate notes, derived from these seminars, are simply entitled *Class Field Theory* [18]. They are now out of print, but remain a classic work familiar to almost every researcher in the field.

In 1956, Artin took a sabbatical leave from Princeton and returned to Germany for the first time since his departure under unhappy circumstances in 1937. He spent one semester at Göttingen and another at Hamburg. While at Göttingen he gave a beautiful set of lectures on algebraic number theory. The notes from this course, written by George Würges, were distributed by the University of Göttingen [14]. They are included in this book.

While in Germany, Artin decided to move back permanently. In 1958 he accepted a Professorship at the University of Hamburg. There he remained for the rest of his life. He died suddenly and unexpectedly on December 20, 1962. He had been given only four years to enjoy his new life at the university of his youth.

There does not exist a full-length biography of Emil Artin. After his death a number of articles on his life and work appeared. Some of them are listed in the bibliography at the end of this introduction [20, 21, 22, 25, 28]. The articles of Richard Brauer [20] and Hans Zassenshaus [28], prominent mathematicians who knew Artin well, are especially worth reading. Zassenhaus had been his graduate student, obtaining his Ph.D. in 1934. The article by J. J. O'Connor and E. F. Robertson [25] is very informative. A delightful and useful resource is the section on Hilbert's ninth problem in *The Honor's Class* by Ben H. Yandell [27], especially pages 230 to 245. The biographical material there is based on, in part, interviews with Natasha (Artin) Brunswick and Artin's children, Michael, Karin, and Tom. There are also a number of interesting and evocative photographs.

The picture that emerges from all these works is that of an exceptionally deep and talented man with manifold interests in art, music, science, and literature, in addition to his profound and abiding love of mathematics. Whatever he did, he did with passion. He was an excellent musician, playing the flute as well as a number of keyboard instruments. He was also a serious amateur astronomer who built his own telescope, grinding the mirror to a near-perfect parabolic shape[1]. All these characteristics might exist in a very inward-directed individual, but Artin loved to communicate with others. He was well known for the excellence of his teaching. Both in lectures and in books he was a master at communicating his vision, his special way of seeing his subject. He exerted tremendous influence in mathematics, both through his creative output and his communicative skills.

It is worth quoting Artin himself (from a 1953 book review of Bourbaki's *Algebra*).

> We all believe that Mathematics is an art. The author in a book, the lecturer in a classroom, tries to convey the structural beauty of mathematics to his readers, to his listeners. In this attempt, he must always fail. Mathematics is logical to be sure; each conclusion is drawn from previously derived statements. Yet the whole of it, the real piece of art, is not linear; worse than that its perception should be instantaneous. We have all experienced on some rare occasions the feeling of elation in realizing that we have enabled our listeners to see at a moment's glance the whole architecture and all of its ramifications.

This quote is given in the article of Brauer [20]. Brauer prefaces Artin's quote with these words of his own.

> There are a number of books and sets of lecture notes by Artin. Each of them presents a novel approach. There are always new ideas and new results. It was a compulsion for Artin to present each argument in its purest form, to replace computation by conceptual arguments, to strip the theory of unnecessary ballast. What was the decisive point for him was to show the beauty of the subject to the reader.

Part 2

Gathered together in this volume are a number of examples of Artin's expository excellence. Three of these are short books: *The Gamma Function*, *Galois Theory*, and *Theory of Algebraic Numbers*. They are each expository in nature, but they all provide examples of Artin's deep feeling for the beauty of his subject, as well as his originality and unique point of view. In addition to these books, we have included a number of papers. Most of these are research papers, but we have felt justified in including them because in these works the writing is so clear, elegant, and insightful. Moreover, the selected works do not make great demands on the background of the reader. For most of them, it suffices to know the material that a good mathematics major learns at a good university.

A word of warning is in order. Artin was a great artist. His writings are not easy to fully appreciate at first glance. The true value of his work becomes apparent only after a certain amount of effort is devoted to its study. What is true of everything presented here is that such effort will be greatly rewarded!

[1]Karin Tate believes the adjective "perfect" would be more appropriate. For the story behind this, see the endnote at the conclusion of the introduction.

The Gamma Function [16] is a translation by Michael Butler (1964) of the German original *Einführung in die Theorie der Gammafunktion*, which appeared in 1931. As Edwin Hewitt asserts in his editor's preface, "... it has been read with joy and fascination by many thousands of mathematicians and students of mathematics." Artin wrote the book to fill a gap he perceived in the literature. The gamma function is important in many parts of mathematics, yet it is often treated as a mere side issue in many beginning books on analysis. In this small book, he develops almost all of the useful properties of the gamma function using nothing beyond elementary calculus. A novel feature is the extensive use made of the notion of log convexity. This approach is due to Harald Bohr and Johannes Mollerup.

Galois Theory [6] is the result of many years of thought devoted to the foundations of the subject. In the thirties, the proof of the main theorem of Galois theory depended on the primitive element theorem. If L/K is a finite, separable extension of fields, the primitive element theorem asserts that L contains an element θ such that $L = K(\theta)$. According to Zassenhaus [28], Artin did not like this state of affairs at all. "He took offense at the central role played by the existence of a primitive element ...". He finally came up with a new proof that was heavily dependent on elementary linear algebra (Part 1 of his book is a condensed treatment of the necessary linear algebra). Primitive elements play no role at all. A novel feature is the use of mappings from a group to a field as elements of a vector space. The new approach did far more than eliminate the use of primitive elements. It led to many new developments, such as the creation of a non-commutative Galois theory.

It must be admitted that although Artin's book *Galois Theory* is beautiful, it is also austere. It is a highly abstract treatment which presaged the use of high abstraction throughout mathematics from the fifties and beyond.

The Theory of Algebraic Numbers [14] is based on lectures given by Artin during his sabbatical semester at Göttingen in 1956–57. The notes were taken by Gerhard Würges and translated and distributed by George Striker. In the late fifties and early sixties one could get a copy by sending $2.50 to George Striker in Göttingen. The book is an introduction to the theory of algebraic numbers using the methods of valuation theory. It provides all the basic results up to and including the finiteness of class number and the Dirichlet unit theorem. The exposition is heavily influenced by the paper *Axiomatic Characterization of Fields by the Product Formula for Valuations* [9], which he wrote with G. Whaples in 1945 (see below). Although not as polished a work as each of the two books discussed above, this is a charming and readable introduction to algebraic numbers as seen through the eyes of a master. A notable feature is the variety of calculations worked out in detail. Known for his love of abstraction, Artin also liked very much to work out detailed examples. Although not too evident in his published works, this tendency is on display here.[2]

The first paper we discuss is the joint paper with G. Whaples mentioned above. After briefly reviewing the theory of archimedean and non-archimedean valuations, the authors point out that, when properly normalized, the set of all valuations of both number fields and function fields satisfy a product formula. This is easily

[2]There are a few errors in the original manuscripts of [6] and [14], which are corrected in the present edition. *Editor's note.*

established. The point of the paper is to show that, conversely, the product formula only holds in these two cases. One rather weak axiom in addition to the product formula is necessary for the proof. With another reasonable axiom, the function field case is shown to hold if and only if the constant field is finite. Number fields and function fields in one variable over a finite constant field are exactly the fields for which class field theory is known to hold. This paper shows that, in some sense, the product formula lies at the base of this imposing structure. Another interesting feature is that the authors deduce the finiteness of class number and the Dirichlet unit theorem (for S-units) without resorting to Minkowski's theorem about lattice points. The paper also points the way to the later systematic use of ideles and adeles in class field theory and beyond. We also include a follow-up paper, wherein the authors are able to weaken further one of the axioms at the base of the theory.

We now come to a set of three papers titled *Kennzeichnung des Körper der reellen algebraischen Zahlen* [1], *Algebraischen Konstruktion reeller Körper* [3], and *Eine Kennzeichnung der reell abgeschlossenen Körper* [5]. The last two were written jointly with Otto Schreier. All three concern the theory of real fields and real closed fields. They are extremely interesting in their own right, but it is worth noting that the material in the first two led Artin to his solution of Hilbert's problem 17 in 1927. We will come back to this after saying something about these papers.

As is obvious from the titles, all three papers are written in German. To make these papers more accessible, we have translated them into English. The translations are fairly literal, with an occasional change in the notation. Also, twice, we have substituted a proof using Zorn's lemma for the original proof which uses the well ordering principle (on the grounds that Zorn's lemma is more likely to be familiar to the modern reader). Otherwise, the translations are straightforward.

In the first paper in this series, Artin attempts to find a purely algebraic characterization of the field P of all real algebraic numbers, i.e., the subset of the real numbers consisting of those which are algebraic over $\mathbb{Q}$. The construction of the real numbers involves limits, so he looks for a characteristic property of the real algebraic numbers that is purely algebraic. Such a property should be preserved under isomorphism. He observes that $\Omega = P(i)$ is the set of all algebraic numbers (the real and imaginary parts of an algebraic number are both algebraic). The paper is then devoted to proving that if $K \subset \Omega$ is a proper subfield of finite codimension, then $\Omega = K(i)$. Moreover, there is an automorphism of Ω that takes K onto P. This is the algebraic characterization sought for; up to isomorphism P is the only subfield of Ω of finite codimension. This short paper is a pleasure to read—one beautiful step after another until the goal is achieved.

In the second paper, the authors search for a purely algebraic definition of the notion of a "real field". They settle on a property of the real numbers that Artin used in the paper about real algebraic numbers discussed above. Namely, they say that a field K is a real field if whenever a sum of squares in K is zero, then each summand must be zero. Equivalently, K is real if -1 cannot be written as a sum of squares in K. This is a purely algebraic property. A field is said to be real closed if it is real but possesses no proper algebraic extension field that is also a real field. They introduce the notion of an ordered field and observe that an ordered field must be a real field. More difficult is the result that a real closed field can be ordered in one and only one way (an element is positive if and only if it is a square). They show that if P is real closed, then $P(i)$ is algebraically closed. Artin's previous paper is then generalized as follows. Suppose Ω is algebraically closed and of characteristic zero. Let K be a proper subfield of finite codimension. Then, K is a real closed field and $\Omega = K(i)$. They go on to show that for polynomials over real closed fields, all the standard theorems of calculus continue to hold, e.g., the intermediate value theorem, Rolle's theorem, the mean value theorem, and the theorems of Sturm. In the next section of the paper, it is proven that if K is a real field and Ω is an algebraically closed extension, then there is a real closed field P between K and Ω such that $\Omega = P(i)$. Finally, this is sharpened to show that an ordered field K can be imbedded in a real closed field in such a way that the order on K is induced from the unique order on P. The last part of the paper concerns the Archimedean axiom and generalizations of it, applications of theory of real closed fields to algebraic number fields, and the construction of an ingenious series of examples that cast new light on the main results.

The aim of the final paper in this series is to show that a field K is real closed if and only if it is of finite codimension in its algebraic closure and is not itself algebraically closed. This had already been shown if K has characteristic zero. The main point then is to show that if K has finite characteristic p, then it cannot be of finite codimension in its algebraic closure Ω unless $K = \Omega$. The new ingredient necessary for the proof is the theory of what has come to be known as Artin-Schreier equations. These are equations of the form $x^p - x - a = 0$, where a is an element of the base field K. Any root of such an equation that is not in K generates a cyclic extension field of degree p over K. Conversely, any cyclic extension of degree p is generated by the root of such an equation. In this paper, cyclic extensions of degree p^2 are dealt with as a tower of two Artin-Schreier extensions. The whole theory is a characteristic p replacement for Kummer theory. It was subsequently considerably generalized by Ernst Witt. The algebraic characterization of real closed fields is an important result. However, it is fair to say that the Artin-Schreier theory of cyclic extensions plays a more central role in mathematics than the reason for its invention.

As mentioned earlier, Artin used the theory of real fields to resolve, positively, Hilbert's problem number 17. This concerns rational functions $f(x_1, x_2, \ldots, x_n)$ in several variables with real coefficients (the usual real numbers). Suppose this function has a non-negative value whenever a real n-tuple is substituted for the variables, assuming the function is defined there. Must it be a sum of squares of rational functions of the same type? Artin showed that the answer is yes in his paper *Uber die Zerlegung definiter Funktionen in Quadrate* [4] published in 1927. Forty years later Albrecht Pfister proved an important refinement: the number of squares necessary is at most 2^n (see [26]). It is interesting to note that Pfister's proof uses Artin's result. He shows, if f is a sum of squares, then it is a sum of at most 2^n squares.

Almost everything we have discussed so far belongs to algebra. Artin's mathematical interests were very far-reaching and he made contributions in other areas as well, e.g., in topology. A classical problem in topology is the question of classifying knots. This subject was actively investigated in the 19th century and remains of great interest today. Artin's contribution was to invent and investigate a somewhat simpler notion. In 1925 he wrote a foundational paper, *Theorie der Zöpfe* [2] (in English, *Theory of Braids*). Braids are somewhat simpler objects than knots. Artin was able to give a satisfactory classification procedure for them. Closed braids turn out to be links, disjoint unions of knots. Thus, the theory of braids turns out to be a tool in the study of knots and links. In fact, braids and the associated braid groups turn up in many different areas of mathematics and even in physics. Artin's invention is today a flourishing subject. Perhaps the easiest way to get a feel for what the subject is about is to read Artin's 1950 article *The Theory of Braids*, which appeared in *The American Scientist* [11]. That article, as well as the more technical 1947 article *Theory of Braids* [10], appear in this volume in reverse chronological order. The latter article is definitely a research article, but it is very well written and accessible to anyone who is comfortable with the definition of the fundamental group of a topological space.

For later developments, together with applications to low-dimensional topology, the reader should consult *Braids, Links, and the Mapping Class Group* by Joan Birman [23]. At the beginning of this book, Birman states, "It is a tribute to Artin's extraordinary insight as a mathematician that the definition he proposed in 1925 . . . for the equivalence of geometric braids could ultimately be broadened and generalized in many different directions without destroying the essential features of the theory". The 2002 book of Seiichi Kamada, *Braid and Knot Theory in Dimension Four* [24], provides an up-to-date treatment together with a huge bibliography (996 items). The first fifty pages are devoted to classical braids and links.

We now come to two articles in the area of analysis. The first, *On the Theory of Complex Functions* [7], concerns the foundations of complex analysis. It presents a very general version of Cauchy's theorem and the Cauchy integral formula based on a novel approach to homology theory via winding numbers. In the introduction to his famous graduate-level text *Complex Analysis*, Lars Ahlfors states that the whole structure of his book was deeply influenced by Artin's insight. Later in the text, he states

> It was E. Artin who discovered the characterization of homology by vanishing winding numbers ties in precisely with what is needed for Cauchy's theorem. This idea has led to a remarkable simplification of earlier proofs.

The second article in analysis that we include here is entitled *A proof of the Krein–Milman Theorem* [12]. This is a letter from Artin to his former Ph.D. student, Max Zorn. The letter was published in the *Piccayune Sentinel* of Indiana University in 1950. This slim newsletter was published from time to time by Zorn himself. It contained news about mathematics and mathematicians. The Krein–Milman theorem states, roughly speaking, that a compact convex subset of a real vector space is the closed convex hull of its extreme points. This basic and useful result is proven by Artin in slightly more than two pages. These pages include the relevant definitions and axioms and all the details of the proof. Needless to say, the version of the theorem presented is very general and the proof is exceedingly elegant.

One bit of explanation is in order. On the second page, Artin uses the phrase "The unmentionable Lemma shows . . . ". This may be puzzling at first. The lemma referred to is Zorn's Lemma, the ubiquitous lemma about partially ordered sets that is discussed in every modern text on graduate algebra.

The last work that is included in this book returns us to the subject of algebra. In it we find an overview of the influence of J. H. M. Wedderburn on the development of modern algebra. This article was published in the *Bulletin of the AMS* in 1950 [13] and is almost as interesting for what it says about Artin's attitudes toward modern algebra as for what it says about Wedderburn. Artin is always the champion of abstract methods and this comes across clearly as he traces the history of the theory of associative algebras. After stating Wedderburn's most famous result, that a simple, finite-dimensional algebra is a full matrix algeba over a division ring, Artin proceeds to present a beautiful generalization due to Claude Chevalley and (independently) to Nathan Jacobson. This generalization is known today as the Jacobson density theorem. After that he discusses another celebrated theorem of Wedderburn: every finite division ring is a field. He relates how this led him to conjecture that a polynomial with coefficients in a finite field with no constant term and having degree smaller than the number of variables must have a nontrivial zero. Chevalley proved this conjecture in 1935, shortly after it was made. Artin went on to conjecture other results of a similar nature for p-adic and global fields. These conjectures led to interesting research by Lang, Ax, Kochen, and others in the 1960s.

This completes our survey of the contents of this book. For readers who want to read more of Artin's expository work, we suggest consulting "The Collected Papers of Emil Artin" [17], the beautiful and valuable book *Geometric Algebra* [15], and the less well known, but worthwhile *Introduction to Algebraic Topology* (written with Hel Braun) [19].

Endnote: It was pointed out earlier that Artin was a serious amateur astronomer who built his own telescope, grinding the mirror to "a near perfect parabolic shape". Karin Tate believes the adjective "perfect" would be more appropriate. To back this up, she related the following charming story. "Of course nothing is perfect. It's just that to make the surface parabolic after grinding out a spherical section certain critical moves have to be made with the grinding stone, and I remember with amusement my father's trying over and over to achieve this. He ground the mirror in our basement, and each time he tried to get a parabolic surface he would do an elaborate test with candles. Mike and I were fascinated by this, and he would tolerate our presence only if we stood still. "Don't move the air!" he would say. Many attempts later, my mother suggested that maybe it was too cold in the basement and that he should bring everything upstairs and try again. My father did this, muttering that he didn't think it would work, but that he'd try it anyway. Of course, it did work, much to my mother's glee. So that is why I made the comment I did - to my father, the mirror was perfect!"

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