Introduction

1. Gentzen's Accomplishments

> Structural proof theory studies the general structure and properties of mathematical proofs. It was discovered by Gerhard Gentzen (1909-1945) in the first years of the 1930s and presented in his doctoral thesis "Untersuchungen über das logische Schließen" [Investigations on logical inference] in 1933.[1]

> The setting of this task is neutral; it does not commit Gentzen to any specific view on the foundations of mathematics, be it formalism, intuitionism, or Cantorism. Gentzen presented first a general theory of the structure of mathematical proofs.[2]

> The object of logic, in Gentzen's view, is to study the general structure of proofs. It is a complete break with the logicist tradition of Frege, Peano, and Russell that Hilbert and his school had been pursuing and in which the notion of logical truth is basic. Proofs that follow a precise set of rules are called derivations, to distinguish them from most of the informal proofs found in mathematics.[3]

A more traditional view:

> Gerhard Gentzen's work in the 1930s (see Gentzen 1969) has been the most influential for the development of modern reductive proof theory in practice... The aim of the Gentzen-Schütte-Takeuti line of development is what I call the constructive consistency proof rationale for reductive proof theory..."[4]

But Gentzen went further than this, leaving behind the constraints of the Hilbert programme. The "true beginnings" of structural proof theory

> may be dated from the publication of the landmark paper Gentzen (1935).[5]

> Interest in proofs as combinatorial structures in their own right was awakened... Nowadays there are more reasons... for studying structural proof theory. For example, automated theorem proving implies an interest in proofs as combinatorial structures; and in logic programming, formal deductions are used in computing.[6]

Indeed Gentzen-style systems and his ideas of natural deduction and sequent calculi dominate proof theory and very strongly influence computer science today. As Dirk van Dalen puts it, "Before Gentzen, proof theory was 'hacker's paradise'";

[1] Sara Negri and Jan von Plato, *Structural Proof Theory*, Cambridge University Press, 2001, p. xi.

[2] Sara Negri and Jan von Plato, "Hilbert's last problem", *Arkhimedes* **2002**, no. 5, p. 4.

[3] Jan von Plato, "Proof theory of classical and intuitionistic logic", L. Haaparanta, ed., *History of Modern Logic*, Oxford University Press, Oxford, in press, pp. 2*ff.*

[4] Solomon Feferman, "Does reductive proof theory have a viable rationale?", p. 4 of the Internet version. *Erkenntnis* **53** (2000), 317–332.

[5] A.S. Troelstra and H. Schwichtenberg, *Basic Proof Theory*, Second Edition, Cambridge University Press, Cambridge, 2000, p. ix.

[6] *Ibid.*

after Gentzen, who was stimulated by Gödel's work, there was "a method full of beauty and elegance."[7]

In Germany especially it was difficult for logicians to appreciate Gentzen's originality because the reception of his work was too often influenced by ignorance and the familiar perspective of Hilbert's programme and Paul Bernays' convenient assessments. In §2.2.1 of Gentzen (1936), Gentzen writes, "The theory of arbitrary mathematical proofs as objects is called 'proof theory' or 'metamathematics'", and he shows what is actually provable after Gödel's results.

Dirk van Dalen describes it with the following words:

> Gentzen was an exponent of the new generation in every regard. A generation which had begun to understand that Gödel wasn't the end of logic but meant the beginning of a new and rich life of the subject. Gentzen found himself at the summit of the discipline as he brilliantly created methods to put into practice in logic, namely those of natural deduction and sequent calculi. With the help of these methods he saw chances to do what was still to be done after Gödel. He thoroughly investigated the problem of excluding contradictions from arithmetic. The adjective "brilliant" is correct here... Gentzen, who without fuss had relegated Hilbert's age to the museum, was immature in his socio-political development.[8]

2. Aims of My Life Story of Gerhard Gentzen

This is the story of the outer life of the mathematician and logician Dr.habil. Gerhard Gentzen; it is not a book about proof theory or its development.[9] Gentzen belongs to the pioneers and founders of proof theory and is one of the "classics"[10] of the field. Although this biography is a contribution to the history of mathematical logic under National Socialism, it is only partly a history of mathematics during these times.[11]

This book does two things: My biography explains Gentzen's ideas and theorems to the layman and gives a full account of the life and tragic death of Gerhard Gentzen. The biography also clarifies Gentzen's position as a mathematician under National Socialism. However, even for those selectively interested in the history

[7] Dirk van Dalen, "Ein Logiker unter den Nazis", pp. 30*ff.*, in *Mitteilungen der Deutschen Mathematiker-Vereinigung*, 1/2003.

[8] *Ibid.*

[9] My work concentrates on the life of Gerhard Gentzen and is not a concise analysis of his work, although it does give indications. A first approach to Gentzen's mathematical theory can be found in Jan von Plato's contribution to this volume or in Gentzen's works themselves. Whoever wants to incorporate the technical work into his reading of this volume is referred to Gentzen's "Untersuchungen über das logische Schließen" (1935) and "Die Widerspruchsfreiheit der reinen Zahlentheorie" (1936). A very good introduction to structural proof theory with all necessary methods and results is given by Sara Negri and Jan von Plato, *Structural Proof Theory*, Cambridge University Press, Cambridge, 2001.

[10] "Classics" are authors about whom, according to Robert Darnton, at least two biographies must be written: a glorification and an unmasking, for a true classic must first be someone who has something to hide. In diametric opposition to readings on the life and work of Gottlob Frege, I could have completely entered without a philology of suspicion, because Gentzen's life, so far as it is known to us, lies open, straight and coherent before us. Who wants to see something else must squint, restrict himself methodically, write from emotion, resentment or rancour—or simply be prepared to be evil.

[11] For a history of mathematics and mathematical logic in this period, see Sanford L. Segal, *Mathematicians under the Nazis*, Princeton University Press, Princeton, New Jersey, 2003. For German language literature on the period, cf. Chapter 4 of the present book.

of mathematical logic under the Nazis, the mathematical and logical position of Gerhard Gentzen within a totalitarian system is also worth reading.[12]

Mathematical logic had political implications under National Socialism, whether it was deemed too formal and lacking of substance by some or recognised as a serious contribution to mathematics by others. The Nazis, in the form of certain functionaries and organisations, had individual representatives who supported, tolerated and promoted mathematical logic and its representatives in individual cases. Gentzen was one of these latter. He enjoyed the protection of Ludwig Bieberbach and the "Deutsche Forschungs-Gemeinschaft" (DFG) [German Research Council], as did the Münster school surrounding Heinrich Scholz (Hermes, Schröter, Behmann, Bachmann, Schweitzer, Ackermann, Kratzer).

Hilbert's programme was the unifying theme among German mathematicians despite the destruction of the Göttingen school by the Nazis. Almost all mathematicians shared Hilbert's vision of a unified mathematics, whether they were Nazi supporters or opponents of Nazism. I don't know any single mathematician who wanted to introduce, for example, intuitionism as an official fundamental philosophy of mathematics of the Nazis, as Weyl, Blumenthal or Menger had in the Weimar Republic. Furthermore the motivation of the Nazis for the protection of mathematical logic by Ludwig Bieberbach and others is still unexplained. Perhaps it was the desire of the Nazis for international recognition and because this field was one with international participation. Modern German mathematics stems from Klein, Hilbert and many other mathematicians of the 19th century, many of whom had in the meantime been driven away to exile or even been killed. The iron resolve of the Nazis to present the highest mathematical rationality as "German mathematics" to the rest of the world failed primarily because of the indignation of the mathematicians who survived exile and execution. There was no room for unwanted academic disputes that would disturb the efficiency of war-related work like cryptography, which was performed by mathematicians as politically diverse as Teichmüller, Witt, and Hasenjæger.[13]

3. Mathematical Logic and National Socialism: The Political Field

The founder of the journal *Deutsche Mathematik*, Ludwig Bieberbach, supported mathematical logic and even used his own funding by the DFG to promote the corresponding logical plans of Heinrich Scholz. Why? Ludwig Bieberbach, the author of the notorious essays "Persönlichkeit und mathematisches Schaffen" [Personality and mathematical creation] and "Stilarten mathematischen Schaffens" [Styles of mathematical creation] defended the academic research in mathematical logic against attacks in the name of "Volk" and race. This requires clarification.

---

[12] On the history of mathematical logic under National Socialism (and the period preceding) there has till now been only the concise observations of Christian Thiel, "Folgen der Emigration deutscher und österreichischer Wissenschaftstheoretiker und Logiker zwischen 1933 und 1945", *Berichte zur Wissenschaftsgeschichte* 7 (1984), pp. 227-256 (here: pp. 248-252). My book only goes as deep technically as is still possible for a mathematically interested layman to follow. The specialists are possibly familiar with the deeper underlying ideas; these are expanded upon here in reviews and contemporary reports of discussions, so that one can form a more solid picture.

[13] Cf. F.L. Bauer, *Entzifferte Geheimnisse. Methoden und Maximen der Kryptologie*, 2nd expanded edition, Springer Verlag, 1997.

> Natural scientists like to live with the illusion that their enterprises could really prosper only in free and democratic societies. The history of science in totalitarian societies should be merely a story of the oppression of "good" science or of one barely surviving in more-or-less unmolested niches. This idealised and ahistorical picture of science falls apart if one considers the crimes, transgressions and "achievements" that came about in the name of national socialist science. (Thomas Weber)

In mathematics it is not absolutely so. Also in the name of mobilisation or self-mobilisation for a political ideology, technically new mathematics is possible.

Mathematical logic was promoted as a part of university mathematics to some extent, and internationally recognised mathematicians researched in the area, though with disgusting interference by the Nazis Dingler, Steck, Thüring and Müller. The mathematical logicians delivered results despite the personnel, financial and organisational "thinning out" of science by the Nazi Party. A serious science was also possible under National Socialism. However, this only succeeded with a kind of job sharing. Some representatives (H. Scholz, H. Hasse, W. Süss and others) professionalised themselves as science organisers and undertook to negotiate with the Nazi Party and its ministers about the conditions for this science. Some representatives (Bieberbach, Vahlen, and others) identified themselves directly with the party, while some overshot the aim entirely (W. Süss) or tried to keep themselves in a kind of balance (H. Hasse). It isn't always and only a fault of the individuals and their individual choices; it might be a fault of the discipline and its organisation as well.

I show the life of the mathematician Gentzen in important details. To be able to judge the mathematics of this time still requires many detailed studies. I have shown a life from the "files" and some testimonies. By the choice of the topics, the fortuity of the documents which came into my hands, I have described the outer life as reliably as I could. Gentzen's life has, I am certain, nonetheless been lived quite differently. When I cite from Gentzen's works and the reviews of his time, I try to show the field of arguments which were around this time to get to know the arguments for and against certain topics.

The history of mathematics isn't even recognised as a disciplinary task, let alone tackled as an interdisciplinary research project. The examination of the mathematics using means and methods of other sciences or humanities is still disgusting for many.

The history of mathematics—as the late Nikolai Stuloff put it—states propositions which occasionally generate different opinions. It is possible that statements are made which cannot be categorised into right or wrong as in mathematics itself. An essential component of the historical work in the history of mathematics is the interpretation and explanation of texts, namely sources and secondary literature. A mathematical statement, once proved, seems timeless. The knowledge of a "historical prelude" which has led to this sentence must not necessarily be known for the understanding of the sentence. In the history of mathematics you will not find a theory, once formulated coherently and proved well, which ever had to be seen as wrong afterwards. But in physics one who treats the laws of falling bodies formulated by Galileo does not need to know at all the laws of falling bodies written down by Aristotle. One who teaches the conception of the heavens of Copernicus and Kepler doesn't have to know the wrong conception of Claudius Ptolemy. However, in mathematics there are no such wrong theories. Take for example the theories of

Archimedes and Apollonius. If the books are no longer learned, it is not because they are wrong but because their methods are simplified today. Their teachings are still valid today. One could still learn mathematics today in substance in the writings of Euclid, or Pappus or Diophantus. This is why one procedure in the history of mathematics is to put the value on the abstract, universal and history-independent validity of mathematics and to the inner development of the concepts and the discipline as a whole. Biographies in this concept of the history of mathematics are still like the hagiographies of elder churchmen or classical artists. The other group of historians emphasises that mathematical production or effective construction takes place within the cultural context, society and the communication codes of the mathematical community and therefore is determined by complex structures, conditional access and certain transmissions. It should be clear that there isn't any conflict between the two groups. "Conceptual Historiography" is expanded by "Contextual Historiography" and their mutual interaction. One who investigates the development of mathematical concepts will have to make himself familiar with the results and methods of disciplines like anthropology, ethnology, philosophy, history, religion or psychology. Is there an interaction between mathematics and the history of ideas? Mathematical sentences are "timeless", but the concepts, even some of their words, in which the theory is formulated are "children of their time". Mathematics is a realm of liberty. So simplifications can be made many years after the justification of a lemma, and still undreamt coherences might be seen then, be formulated and taken more abstractly than the inventor might have ever imagined. Although the eternal truth of proved sentences is valid, some theories could often be formulated afterwards in a better, more elegant way and within applied sciences. I do hope I am making a small and fair contribution to the historiography of modern logic through the biography of Gerhard Gentzen. A bright and conceivable history of modern logic isn't understandable without one's biography using conceptual and contextual ideas. Moritz Epple (1999) writes: "It is only neither all about a story of mathematical concepts and ideas nor all about a collection of socio-biographical footnotes. . . for the row of the mathematical results. Rather it should be made clear at least at some important episodes, how mathematical knowledge produces and was used in concrete historical courses of action." (p viii.) Moritz Epple doesn't always succeed in it either, which is why he also includes socio-biographical inserts in his texts and theses. My biography shall nevertheless support the aim envisioned by Epple in the field of proof theory because, without clarification of historical facts, the different forms of evolving mathematical treatments, methodical and resulting knowledge, and epistemic configurations or its reflection are not once meaningfully describable.

Therefore there is—standing completely unconnected beside each other—next to the problem of history of mathematics[14] now a “contextual history of mathematics”.[15] I myself feel somewhat obligated to the German tradition,[16] which is embedded in the world history of mathematics.[17]

---

[14]Morris Kline, *Mathematical Thought from Ancient to Modern Times*, Oxford University Press, New York, 1972.

[15]Ronald Calinger, *A Contextual History of Mathematics to Euler*, Prentice-Hall, Upper Saddel River, 1999.

[16]New exemplary literature in the German language: Herbert Mehrtens, *Moderne. Sprache. Mathematik*, Suhrkamp, Frankfurt am Main, 1990; Moritz Epple, *Die Entstehung der Knotentheorie. Kontexte und Konstruktionen einer modernen mathematischen Theorie*, Vieweg, Wiesbaden, 1999.— Craig Smoryński (1988, p. 16) once wrote: “Philosophy of mathematics was briefly concerned with foundations. Nowadays, philosophers are mostly interested in the philosophy of mathematics as a testing ground for their epistemological theories, and most mathematicians would find it pretty boring stuff.” In the meantime the history of mathematics has also become a sort of testing ground. Perhaps this has an end if the “descriptive phase” (C. Smoryński) is completed at least in the field of the newer mathematical logic of the 20th century. As long as this isn’t the case, my ideal lies between Jan of Plato, *Creating Modern Probability. Its Mathematics, Physics and Philosophy in Historical Perspective*, Cambridge University Press, Cambridge, 1994; and Dirk Van Dalen, *Mystic, Surveyor, and Intuitionist. The Life of L. J. Brouwer. Vol 1: The Dawning Revolution* and *Vol. 2: Hope and Disillusion*, Clarendon Press, Oxford, 1999 and 2005. Here it shows how splendid it is when two trained scientists write professional science history. The excellent use of the history of mathematics in a textbook for working mathematicians is shown in exemplary fashion by Smoryński in *Logical Number Theory*, vol. 1 (Springer, New York, 1991) and Vol. 2 (to appear). For all three a dictum of Hermann Weyl holds: “A scientist who writes on philosophy faces conflicts of conscience from which he will seldom extricate himself whole and unscathed; the open horizon and depth of philosophical thoughts are not easily reconciled with that objective clarity and determinacy for which he has been trained in the school of science” (p. v, *Philosophy of Mathematics and Natural Science*, Princeton University Press, Princeton, 1949).

[17]Joseph W. Dauben and Christoph J. Scriba (eds.), *Writing the History of Mathematics: Its Historical Development*, Birkhäuser, Basel, 2002.