Preface to the English Edition

In revising this book for translation I have cut down on the polemics, and I have incorporated new facts and material and what I have learned in these years, notably from some reviews of the German edition—especially the one of Dirk van Dalen—as well as from continual discussion with Jan von Plato and Craig Smoryński. I have taken a stand which is less critical of tradition, but more thorough acquaintance with what Gentzen and proof theory have done has pushed the concept of Gentzen as a "follower of the Hilbert Programme" further into the background. A clear exposition, description and interpretation of Hilbert's Programme by Craig Smoryński in appropriate style, language and richness of ideas can be found in this book. Gentzen's work should decisively not just be seen as contributing only to the post-Gödelian development of Hilbert's Programme. It this were true, Gentzen would be of almost no interest at all today to creative working mathematicians, but only a concern for antiquarians.

Gerhard Gentzen (1909-1945) is the founder of modern structural proof theory. His permanent and sustained methods, rules and structures have resulted not only in a technical mathematical discipline called "proof theory" but also in applications in computer science (e.g. program verification) and all kinds of effective mathematics. Someday the performance of the pioneer Gerhard Gentzen will be seen like the creations of Heisenberg, Schrödinger or Dirac. Gentzen's natural deduction, sequent calculi and ordinal proof theory certainly impress with their appearance, clarity and elegance. And, the techniques he developed in the years from 1931 to 1939 are now the minimum standard in proof theory.

Gentzen wrote Bernays in a letter dated 3 March 1936:

> I also don't know if I can claim any "priority" in all particulars; I wish only this once to survey further work on the programme from the now established points, the carrying out of which admittedly can require some years or decades.

And this programme was decisive for the post-Gödelian period of logic.

Following Jan von Plato, we recognise three stages in Gentzen's ideas: the conviction that the consistency of arithmetic can be proven constructively arose from his manuscript "Über das Verhältnis zwischen intuitionistischer und klassischer Arithmetik" [On the relation between intuitionistic and classical arithmetic] withdrawn in March 1933. He proved this consistency in 1935 in his "Die Widerspruchsfreiheit der reinen Zahlentheorie" [The consistency of pure number theory]. And in his habilitation thesis submitted in 1939, "Beweisbarkeit und Unbeweisbarkeit von Anfangsfällen der transfiniten Induktion in der reinen Zahlentheorie" [Provability and unprovability of initial cases of transfinite induction in pure number theory] he proved directly the unprovability of ε_0-induction in Peano Arithmetic. And he achieved all this on the basis of a calculus he constructed for such tasks in

1932/1933. Even Paul Bernays agreed in 1938 that Gentzen had a place in foundational research in his own right, one that did not fall under the banner of "Hilbert's Programme". Jan von Plato describes the actual development of Gentzen and proof theory in detail in this book.

But though a good number of passages have been revised, and though there are some small rearrangements in the order of treatment, the book has substantially remained the same. So those "innocent" of logic or proof theory should find it understandable.

I cannot claim to have provided a definitive history of proof theory in the thirties and forties of the last century in Nazi Germany or a complete account of Gentzen, the man and the genius. Still I trust that this book gives a full and perspicuous presentation of the evidence and thus will be useful even to those who are not inclined to draw the same conclusions from it.

My special thanks are to Craig Smoryński, who not only completed the laborious task of translation in a spirit of most pleasant and full collaboration in Westmont and different places in the world, but to whom is due the initiative and diligent working out of an improvement of ideas, content and style for the English edition. Everybody who knows his ideas will agree that they shed light on my dark thoughts. The responsibility for the content and for all that may be wrong with it, however, remains mine.

The Department of Philosophy of the University of Helsinki has aided Craig and me with a visit where Craig, Sara Negri, Jan von Plato and I again could meet and discuss various aspects of Gentzen and his work. Some parts of this book were written in the beautiful atmosphere of the Mathematisches Forschungsinstitut of Oberwolfach. I acknowledge all this favour as a factor in the excellence of this book. The preparation of this edition was done under ideal working conditions.

I hope that this book will be judged as a contribution to the philosophy and the history of logic and proof theory and not merely to the art of biography of mathematicians.

This book is dedicated to Heidrun Trott and my children, Jacob Menzler, Laura Trott and Gwendolyn Trott.

<div style="text-align: right">Eckart Menzler-Trott</div>

Munich, 1 June 2006

Dr. Christian Thiel has let me examine the material of Gentzen's Nachlass, so far as it has been deciphered, and constructed a painful list of my literal and typographical errors from the German and the present edition of the book. Contentual and formal criticism amounting to several pages were sent by Dr. Reinhard Siegmund-Schultze, and almost all of his suggestions have been taken into account in this edition. The reviews of the German edition by Dirk van Dalen, Petr Hájek, Ivor Grattan-Guinness, Albert C. Lewis, Rudolf Taschner, Volker Peckhaus, Peter Schreiber, and many others were very helpful, and I have been able to put some of their stimulation to good use here.

Some material—items of information or suggestions—which I acknowledge at the appropriate places in the book, have been provided by: Dr. Norbert Schappacher, Dr. Justus Diller, Dr. Herbert Mehrtens, Jens Erik Fenstad, Dr. Volker R. Remmert, Dr. Harold Schellinx, Dr. Milan Vlach, Georg Kreisel, Gerd Robbel, Dr. Hans Hermes, Jan von Plato, Dirk van Dalen, Dr. Enno Folkerts, Dr. Gisbert Hasenjaeger, Dr. habil. Renate Tobies, Dr. F. L. Bauer, Ulrich Bieberbach, Dr. Günther Engler, Dr. Gerhard H. R. Reisig, Dr. Moritz Epple, Dr. Hans Rohrbach, Dr. Premysl Vihan, Dr. Olga Kraus, Dr. Ina Kersten, Jürgen Kasüske, and some others.

I learned a lot about Gentzen's logic from the writings of Wolfram Pohlers, Wilfried Buchholz, Helmut Schwichtenberg, Grisha Mints, Michael Rathjen, Sol Feferman, Wilfried Sieg and many others and thank them for their personal communications.

My thanks go to the libraries and archives: Petra Blödorn-Meyer (Handschriftenabteilung der Staats- und Universitätsbibliothek Carl v. Ossietzky Hamburg); Dr. Laetitia Boehm (Archiv der Ludwig-Maximilian-Universität München); W. Schultze (Archiv der Humboldt Universität Berlin); Dr. Beat Glaus and Ms. Flavia Lanini (Archiv der ETH Zürich); Dr. Ulrich Hunger (Universitätsarchiv Göttingen); Dieter Speck (Universitätsarchiv Freiburg); Miroslav Kunstat (Archiv Univerzity Karlovy Prag); Frau H. Bertram (Mathematisches Institut Giessen); Berlin Document Center, Bundesarchiv Koblenz; Institut für Zeitgeschichte; Mathematisches Institut Göttingen; Deutsche Dienststelle; Militärarchiv Freiburg; Institut für mathematische Logik und Grundlagenforschung Münster; Bayerische Staatsbibliothek; Dombibliothek Freising; Hessische Staatsbibliothek Wiesbaden.

For their constant support for decades I particularly and very warmly thank Antiquariat Kitzinger in Munich's Schellingstraße and Antiquariat Renner, who have always gone out of their way to procure all books which on account of the consequences of war or chronic underfunding are not available in German libraries.

Quick Note from the Main Translator. An old set of guidelines from the American Mathematical Society declared that the requirements for translating mathematics into English were, in order, a knowledge of the mathematics in question, a knowledge of English, and a knowledge of the language of the original. I have read mathematics and biography in German without any major difficulties in the past, but the present book has incredible breadth. The philosophical quotations of the National Socialists in Chapter 4, in particular, proved too much for me and I enlisted the aid of my friend Ed Griffor, whose knowledge of philosophy and the German language far exceeded my own. I owe him a debt of gratitude.

As I was learning LaTeX simultaneously with translating this book, the typography may not be up to par. For this I apologise to Eckart and the reader.

    With respect to my appendix on Hilbert's Programme, I should like to take the opportunity here to express my indebtedness to Dirk van Dalen, both for general stimulation and particular information on Brouwer and Hilbert.

Craig Smoryński

Westmont, 17 April 2007