Preface

A semigroup is a set that is closed under an associative binary operation. We might therefore regard a semigroup as being either a defective group (stripped of its identity and inverse elements) or a defective ring (missing an entire operation). Indeed, these are two of the original sources from which the study of semigroups sprang. However, to regard the modern theory of semigroups simply as the study of degenerate groups or rings would be to overlook the comprehensive and independent theory that has grown up around these objects over the years, a theory that is rather different in spirit from those of groups and rings. Perhaps most importantly, semigroup theory represents the abstract theory of transformations of a set: the collection of all not-necessarily-invertible self-mappings of a set forms a semigroup (indeed, a *monoid*: a semigroup with a multiplicative identity), but not, of course, a group. The development of the theory of semigroups from these various sources is the subject of this book. I chart the theory's growth from its earliest origins (in the 1920s) up to the foundation of the dedicated semigroup-oriented journal *Semigroup Forum* in 1970. Since the theory of semigroups developed largely after the Second World War, it might be termed 'Cold War mathematics'; a comparison of the mathematics of semigroup researchers in East and West, together with an investigation of the extent to which they were able to communicate with each other, is therefore one of the major themes of this book.

I believe that semigroup theory provides a particularly good illustration of these problems in East-West communication precisely because it is such a young theory. We are not dealing here with a well-established mathematical discipline, to whose traditions and methods mathematicians in East and West were already privy and had in common when the Iron Curtain descended. Instead, the foundations of many semigroup-theoretic topics were laid independently by Soviet and Western mathematicians who had no idea that they were working on the same problems. Thus, different traditions and priorities were established by the two sides from the earliest days of the theory.

The structure of the book is as follows. In Chapter 1, I set the scene by considering the status of algebra within mathematics at the beginning of the twentieth century. I discuss the coining of the term 'semigroup' in 1904 and give an overview of the broad strokes of the subsequent development of the theory of semigroups.

Chapter 2 is devoted to the major theme mentioned above: the East-West divide in mathematical research. I provide a general discussion of the extent to which scientists on opposite sides of the Iron Curtain were able to communicate with each other and the degree to which the publications of one side were accessible to the other.

The description of the development of semigroup theory begins in Chapter 3 with a survey of the work of the Russian pioneer A. K. Sushkevich. I investigate his

influences, the types of problems that he considered, and the legacy of his work, with an attempt to explain his lack of impact on the wider mathematical community.

Chapter 4 is the first of two chapters dealing with the semigroup-theoretic problems that arose in the 1930s by analogy with similar problems for rings. Thus, Chapter 4 concerns unique factorisation in semigroups, while Chapter 5 deals with the problem of embedding semigroups in groups. In these two chapters, we see how the investigation of certain semigroup-related questions began to emerge, although this was not yet part of a wider 'semigroup theory'. The beginning of a true *theory* of semigroups is dealt with in Chapter 6, which concerns the celebrated Rees Theorem, together with a result proved by A. H. Clifford in 1941, which might be regarded as semigroup theory's first 'independent' theorem: a result with no direct group or ring analogue.

Paul Dubreil and the origins of the French (or, more accurately, Francophone) school of 'demi-groupes' are the subject of Chapter 7, while Chapter 8 concerns the expansion of semigroup theory during the 1940s and 1950s, both in terms of the subjects studied and also through the internationalisation of the theory. I thus indicate the major semigroup-theoretic topics that emerged during this period and also give an account of the various national schools of semigroup theory that developed. Chapter 8 marks something of a watershed in this book: the material appearing before Chapter 8 represents the efforts of the early semigroup theorists to build up their discipline, while that coming after may be regarded as being part of a fully established theory of semigroups.

Chapter 9 concerns the development of the post-Sushkevich Soviet school of semigroup theory through the work of E. S. Lyapin and L. M. Gluskin. I pick up the discussion of Chapter 2 and try to give an indication of the extent to which Soviet semigroup theorists were aware of the work of their counterparts in other countries and of the level of knowledge of Soviet work outside the USSR.

In Chapters 10 and 11, I deal with two major aspects of semigroup theory that emerged in the 1950s: the theory of inverse semigroups and that of matrix representations of semigroups. Both of these remain prominent areas within the wider theory (though the latter was considerably slower in its development), and both furnish us with well-documented examples of the duplication of mathematical research across the Iron Curtain.

In the final chapter, I draw the book to a close by considering the first monographs on semigroups, the early seminars, and the first conferences.

The focus here is upon the history of the *algebraic* theory of semigroups, rather than that of the *topological* theory, which is dealt with elsewhere (see the references on page 10). I have, by necessity, been very selective in the material that I have included here, particularly in connection with the semigroup theory of the 1960s, which is simply too broad to cover in its entirety. A historical account that attempted to cover the whole of semigroup theory would be near-impossible to write and little easier to read. Nevertheless, the book is liberally sprinkled throughout with endnotes in which I give rough indications of other aspects of the theory that are not covered in the main text. One broad area that is perhaps somewhat conspicuous by its almost total absence is the theory of formal languages and automata, together with the Krohn–Rhodes theory of finite semigroups: when choosing which topics to include here, I decided that these theories were simply too large to do justice to within the confines of a book such as this.

I have tried to make this book accessible to as large an audience as possible. Thus, although I have supposed a general familiarity with abstract algebra on the part of the reader, I have not assumed any knowledge of the specifics of semigroup theory. Many elements of the relevant mathematics are introduced as we go along, but some of the more fundamental notions from semigroup theory are summarised in the appendix.

With regard to the notation used throughout the book, I have, as far as possible, retained the notation used by the original authors. Notable exceptions are those few cases where the original notation might prove to be confusing or ambiguous. Some of the authors considered here composed their functions from right to left, while others, following a convention often adopted in semigroup theory, composed from left to right (see the appendix). I have not imposed a uniform direction for the composition of functions but have again retained the conventions of the original authors. Nevertheless, I have endeavoured to make the particular direction clear in each case.

Languages

The presentation of as full a picture as possible of the international development of semigroup theory necessarily involves the use of sources in many different languages. Wherever I have quoted from a foreign source, I have provided my own English translation (unless stated otherwise), together with the text of the original in an endnote. However, I have saved space in the bibliography by only giving the English translations of titles of foreign sources, except for those items in French or German (plus one or two in Spanish and Italian), for which I have only given the original: readers are, I estimate, likely to be able to translate these for themselves. With regard to non-English terminology, I have endeavoured to supply appropriate translations but in every case have provided the original term in parentheses at the point of the translation's first appearance in the text.

Regarding the Latin transliteration of Cyrillic characters, I have, for the most part, adopted the conventions of the journal *Historia Mathematica*. These conventions are summarised in Table 0.1. Notice in particular that the two silent letters, the hard (ъ) and soft (ь) signs, are omitted from transliterations altogether. Indeed, this latter point and the use of i instead of ĭ for й are the only differences between the *Historia Mathematica* transliteration conventions and those employed in *Mathematical Reviews*. I have chosen the conventions in the table purely on the grounds of simplicity and aesthetics. I deviate from these conventions, however, in the cases of names that have commonly accepted Latin spellings. Thus, for example, 'Шмидт' is transliterated as 'Schmidt', rather than 'Shmidt', and 'Вагнер' as 'Wagner', rather than 'Vagner'.

Some of the Soviet authors whom I mention here have had their names transliterated in various ways, according to different conventions. Thus, for example, Сушкевич has appeared as 'Sushkevich', 'Suškevič', 'Suschkewitz', 'Suschkewitsch', and 'Suschkjewitsch', while Ляпин has been rendered as 'Lyapin', 'Ljapin', and 'Liapin', and Мальцев as 'Maltsev', 'Mal'tsev', 'Malcev', and 'Mal'cev'. In some cases, these authors published work under different transliterations of their names; these works have been listed in the bibliography according to the name under which they were published. Thus, for example, A. K. Sushkevich's work appears under both

TABLE 0.1. Conventions for transliteration from Cyrillic characters.

For Russian								
Аа	=	Aa	Кк	=	Kk	Хх	=	Kh kh
Бб	=	Bb	Лл	=	Ll	Цц	=	Ts ts
Вв	=	Vv	Мм	=	Mm	Чч	=	Ch ch
Гг	=	Gg	Нн	=	Nn	Шш	=	Sh sh
Дд	=	Dd	Оо	=	Oo	Щщ	=	Shch shch
Ее	=	Ee	Пп	=	Pp	ъ	=	-
Ёё	=	Ee	Рр	=	Rr	ы	=	y
Жж	=	Zh zh	Сс	=	Ss	ь	=	-
Зз	=	Zz	Тт	=	Tt	Ээ	=	Ee
Ии	=	Ii	Уу	=	Uu	Юю	=	Yu yu
Йй	=	Ii	Фф	=	Ff	Яя	=	Ya ya
Additional letters for Ukrainian								
Єє	=	Ee	Іі	=	Ii	Її	=	Ïï

'Sushkevich' and 'Suschkewitsch', while A. I. Maltsev's is listed under 'Maltsev' and 'Malcev'.

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