

The Soviet Mathematical School

The book now before the reader is devoted to a dramatic episode in the history of mathematics in the USSR, an ideological campaign stirred up in 1936 against the prominent academician Nikolai Nikoalevich Luzin (1883–1950). It marked an important turning point in the history of the Soviet Mathematical School, one of the leading schools of the twentieth century. The school was formed during the 1930s from two earlier schools: the Leningrad School, which had originated in one of the most famous European schools of the last third of the nineteenth century, the brilliant Petersburg School of P. L. Chebyshev (1821–1894), and the Moscow School that grew up at Moscow University on the eve of the First World War.

The leaders of the Moscow School D. F. Egorov (1869–1931) and Luzin were able to plant in the Moscow soil a branch of mathematics that was a recent innovation, the theory of functions of a real variable. This area had been developed during the 1890s by the French mathematicians Emile Borel (1871–1956), René Baire (1874–1932), and Henri Lebesgue (1875–1941). The new school achieved solid recognition by the end of the 1910s. The results of Luzin himself and those of his students D. E. Men'shov (1892–1988), M. Ya. Suslin (1894–1919), A. Ya. Khinchin (1894–1959), and P. S. Aleksandrov (1896–1982) in the field of theory of functions and sets became well-known in Europe and advanced Moscow to the ranks of the most important mathematical centers of the time. The early years of the Moscow School coincided with a time of great turbulence for the Russian state: The First World War, the February and October Revolutions of 1917 that occurred during the war, and finally the Civil War that raged from 1917 to 1921. These events had an extremely negative impact on the educational and research institutions of the country. Despite all that, the development of the Moscow School of the Theory of Functions proceeded apace and with great success.

The research areas of the school soon broadened. New areas were adjoined to the traditional Moscow fields of applied mathematics, differential geometry, and others. One of them was the theory of functions of a complex variable. Luzin himself worked in this area, along with his students V. V. Golubev (1884–1954), I. I. Privalov (1891–1941), Men'shov, and Khinchin. It is in their work that one finds the the beginning of the research area distinguished by the results of M. A. Lavrent'ev (1900–1980) and M. V. Keldysh (1911–1978). Aleksandrov and P. S. Uryson (1898–1924) laid the foundations of the School of Topology, which soon produced A. N. Tikhonov (1906–1993) and L. S. Pontryagin (1908–1988). Khinchin and A. N. Kolmogorov (1903–1987) wrote their first papers in probability theory, which were later to be brilliantly extended both by the authors themselves and by their numerous students. Khinchin's seminar on number theory began to function during the 1925/26 academic year; it formed the foundation of a new number-theoretic school, among whose members were L. G. Shnirel'man (1905–1938) and A. O. Gel'fond

(1906–1968). A School of Functional Analysis began to form in the late 1920s and early 1930s, having as members L. A. Lyusternik (1899–1981), Shnirel'man, Kolmogorov, and A. I. Plesner (1900–1961), who had immigrated from Germany. One of the greatest mathematicians of the twentieth century, I. M. Gel'fand (1913–2009), was a distinguished graduate of this school. On the basis of all this work the range of research in the field of both ordinary and partial differential equations greatly expanded; among the researchers were V. V. Stepanov (1889–1950) and I. G. Petrovskii (1901–1973). Research began in the area of mathematical logic, with I. I. Zhegalkin (1869–1947), Kolmogorov, and later P. S. Novikov (1901–1975).

We have noted here only a few of the important areas that were developed with success in the Moscow of the 1920s and 1930s and whose foundations lay in the research of the Moscow School of the Theory of Functions. If we add to this list the first achievements of the Moscow mathematicians in modern algebra under the leadership of O. Yu. Shmidt (1891–1956), who had moved to Moscow from Kiev, and the traditional areas of Moscow mathematics already mentioned—the applied mathematics pursued by researchers such as S. A. Chaplygin (1869–1942) and differential geometry, studied by Egorov, S. P. Finikov (1883–1964), and V. F. Kagan (1869–1953), who arrived from Odessa and gave a flavor of tensor analysis to the topics studied, one can say that the range of research among the Moscow mathematicians had become extraordinarily broad and embraced at least a significant portion of the mathematics of the time, if not quite all of it.

Such a broadening of the range of research in Moscow, however, was not only evidence of the extraordinary creative potential accumulated in the bosom of the Moscow School, whose leader Luzin continued to be during the 1920s; it also turned out to be the source of a conflict between him and his students. Having grown accustomed to the role of an unquestioned authority among his students, he was very disturbed by their maturation as scholars, their deviation away from the paths he himself had marked out and onto their own way. The story of that conflict, which led to the “Luzin case” will be told below. Right now, we return to the question of the genesis of the Soviet Mathematical School and to the other component of it, the Leningrad School.

This school developed out of the research of the greatest Russian mathematician of the second half of the nineteenth century, Pafnutii L'vovich Chebyshev. This research—especially probability theory, polynomial approximation, differential equations, and mathematical physics—was primarily applied mathematics. The only exception for Chebyshev and his students was number theory, to which Chebyshev had been attracted by the need to edit the number-theoretic papers of Euler, a project that Academician V. Ya. Bunyakovskii (1804–1889) had brought to his attention when Chebyshev was young. The principal areas of research of this school were the number theory of Chebyshev, E. I. Zolotarëv (1847–1878), A. N. Korkin (1837–1908), and A. A. Markov (1856–1922); probability theory, which was studied by Chebyshev, Markov, and A. M. Lyapunov (1857–1918); polynomial approximation, studied by Chebyshev, A. A. Markov, and V. A. Markov (1871–1897); and differential equations and mathematical physics, studied by Chebyshev, Lyapunov, V. A. Steklov (1864–1926), and N. M. Gyunter (1871–1941). The outstanding results obtained by Chebyshev and his students in these fields were highly regarded throughout the mathematical world and established the reputation of Petersburg as a recognized mathematical center.

The applied nature of research at the Petersburg School combined with the positivist attitudes of its leaders, with their refusal to accept idealistic philosophy, to say nothing of religious philosophy, and with their militant atheism. Especially prominent in this respect was A. A. Markov, who became the leader of the school after the death of Chebyshev. This ideological bent of the Petersburg mathematicians contrasted with the attitudes that prevailed among the Moscow mathematicians, which were characterized by antipositivism, a profound interest in idealist philosophy—as a result of which the appellation “philosophico-mathematical” became a standard description of the Moscow School!—and in Orthodoxy. Thus the relations between the schools had a confrontational character, leading to clashes that frequently ended in heated debates at sessions of the Moscow Mathematical Society.

This opposition of the two mathematical centers, academic Petersburg—where the tone was set by the Imperial Academy of Sciences—and the old capital, in whose mathematical circles the University and the Moscow Mathematical Society were dominant, created a tension in the national mathematical community that was in the process of formation. (A significant portion of the professoriate in the provincial universities was made up of graduates of the universities in the two capitals.)

The Petrograd mathematical community suffered more than the Moscow community from the hardships that befell them during the difficult years of the 1917 revolutions and the subsequent Civil War. Nevertheless, by the late 1920s life had begun to settle down even in Leningrad, as Petrograd—which had previously been known as Saint Petersburg and had been renamed during a wave of anti-German sentiment stirred up in 1914 by the war with Germany—was called after 1924. (In 1992, a new wave of reform brought back the original name of Saint Petersburg.) Research of a high caliber was conducted in number theory by I. M. Vinogradov (1891–1983), in differential equations and mathematical physics by Gyunter, V. I. Smirnov (1887–1974), and S. L. Sobolev (1908–1989). The early 1930s saw the first research of L. V. Kantorovich (1912–1986), and S. N. Bernshtein (1880–1968) arrived from Khar’kov in 1933.

Thus by the early 1930s the two schools in Moscow and Leningrad that formed the foundation of the Soviet Mathematical School were both developing actively. The tension between them had relaxed somewhat, although it remained at a fairly high level. To understand what came next, one must bear in mind the socio-political context in which mathematical life was lived.

The first years of Soviet power were a time of complete confusion and uncertainty for science and education. This uncertainty was largely the result of the expectation on the part of the Bolsheviks who had acquired power that a world-wide revolution would begin very soon. In the reasoning of the new authorities, it made sense to undertake the construction of a system of Communist education and the institutions of a new proletarian science only after that revolution was accomplished. Education in the Gymnasium and the old system of higher education, along with the Imperial Academy of Sciences, appeared to the new ideologues to be a legacy of the old regime, and it was time to be done with such institutions. The mid-level schools (Progymnasium, Gymnasium, and Realschule) were to be replaced by a unified vocational school, in which instruction was to be carried out on the basis of new revolutionary principles. The formulation of these principles became

the task of numerous revolutionary reformers, who completely ruined education in a very short period of time. The fact that education continued to take place despite everything is to the credit of the teachers of the old school, who taught in the only way they knew how or considered necessary, that is, the old-fashioned way. In the thinking of the revolution-oriented party functionaries, the place of the Academy of Sciences would be taken by a Socialist Academy that was founded in 1918, one of whose purposes was proclaimed to be the establishment of a new proletarian science built on the basis of the “one true teaching,” Marxist–Leninist philosophy. The position of the Russian Academy of Sciences, which remained in Petrograd when the Soviet government headed by V. I. Lenin (1870–1924) moved to Moscow, remained undecided for a long time. The danger that it might be liquidated as a relic of the old monarchist regime remained. The leading role in the preservation of the Academy and its incorporation into the new Soviet institutions was played by its vice-president V. A. Steklov, a man of leftist convictions who made personal contact with Lenin and was able to convince the Soviet leadership of the importance and usefulness of the Academy in the cause of “building a socialist society.”

After the first ardor of revolution had passed and hope for an imminent worldwide revolution had died, it became clear that the new governmental organization, the Union of Soviet Socialist Republics, which found itself surrounded by countries hostile to the Bolshevik state, would have to make do with only its own resources, which were badly depleted after all the wars and revolutions. Agriculture, industry, mid-level and higher education, science—all lay in ruins. The most perceptive people had already begun to see the gathering clouds of the next war. It was in this situation that I. V. Stalin (1879–1953), who came to power after the death of Lenin, and whose power came to be almost absolute in the 1930s, instituted the policy of collectivization and industrialization of the country. The center of new economy was to be a powerful military industry. Well-trained personnel were needed to fulfill such ambitious plans: skilled workers and engineers, whose numbers were very small. There was a need to build a new system of schools and organize new mid-level specialized schools and institutions of higher learning. Moreover, all this needed to be done in a very compressed period of time in a state of mobilization. For that reason, those at the highest level of authority made the only decision that was appropriate in such a situation: to reconstitute the old system of secondary education, adapting it to the new realities. (For example, the classical mathematical textbooks of A. P. Kiselëv, which essentially went all the way back to Euclid’s *Elements*, were re-introduced into the secondary schools, only now adapted to the current conditions by competent mathematicians. Khinchin directed this project.)

As for the Russian Academy of Sciences, which had now become the USSR Academy of Sciences, in accordance with its 1927 Charter (on whose creation V. A. Steklov had done a considerable amount of work), it was proclaimed the head research institution of the country, whose main task was declared to be socialist construction. In accordance with the plan of Stalin, the “edifice” of Soviet science was to be constructed in the shape of a gigantic pyramid at whose apex the USSR Academy of Sciences—the “headquarters of Soviet science”—was to be located. Of course, the “headquarters” had to be located close to the “boss” of the Soviet state, near to hand for him. For that reason in 1934, the leadership of the Academy and several of its leading institutes, among them the Steklov Mathematical Institute, were moved to Moscow.

This relocation of the “Steklovka” (as the Steklov Institute is customarily called in mathematical circles) was fraught with consequences for the development of mathematics in the country. The leading mathematicians of the two capitals, who had been in a confrontational posture that had become a tradition, were forced to co-operate. As one of the participants in these events, B. N. Deloné (1890–1980), who made the journey from Leningrad to Moscow,¹¹ wrote

... between the Euler–Chebyshev School of Petersburg and the Luzin School of Moscow... there was always so much mutual antagonism that neither understood the other until the Academy was moved to Moscow. After that, we began a rapprochement, and the result of this reconciliation of the two schools was what we now call “Soviet mathematics.”

What took place was the merger of two schools that, although they had common origins, were very different in their outlook. There was a synthesis of the tradition of the Petersburg School of Mathematical Physics of S. L. Sobolev and the Moscow tradition of research in the field of the geometric theory of partial differential equations that originated with K. M. Peterson and was being carried on by Petrovskii; of the Moscow group in functional analysis, represented by Kolmogorov, Lyusternik, and Plesner with the Leningrad group of Sobolev. The Chebyshev approach to probability theory, carried on by his heir S. N. Bernshtein, merged with the Moscow approach that grew up in the context of measure theory developed by Khinchin and Kolmogorov. Two lines of development of number theory merged, the Chebyshev approach followed by I. M. Vinogradov and the new Moscow approach of Khinchin, Gel’fond, and Shnirel’man. There were two lines of development in algebra that originated in the Kiev School of D. A. Gravé—the Moscow line represented by Shmidt and A. G. Kurosh and the Leningrad line represented by B. N. Deloné. From all this a powerful creative potential arose. Such was the genesis of the Soviet Mathematical School.

The events connected with the “Luzin case,” which is the subject of the present book, took place two years after the Academy had moved to Moscow, in the summer of 1936. An *ad hoc* commission of the Academy specially constituted to investigate this “case” was to examine the accusations against Luzin and render a report on his activity. The commission was chaired by the Academy vice-president, the Old Bolshevik G. M. Krzhizhanovskii (1872–1959). (The reorganized Academy of Sciences had been obliged to include among its members some representatives of the new governing authorities). Should this activity be characterized as “hostile” (and it was in this vein that the events of the first few meetings of the commission took place) the question of his expulsion from the Academy might have been raised. The mere fact of having been expelled from the Academy for activities damaging to the Soviet state—that is, in the language of the time, for “disruptive” activity—automatically entailed handing the “case” over to the organs of the NKVD—the National Commissariat of Internal Affairs, heir to the OGPU (Combined State Political Administration) under the Council of State Commissars, which was the brainchild of F. E. Dzerzhinskii (1877–1926). The NKVD was the central state organ for combatting crime and maintaining order in society, as well as maintaining

¹¹Boris Nikolaevich Deloné. Conversation of 14 December 1973, in: *Mathematicians Speak: The V. D. Dubakin Collection of Phono-documents*. Moscow, 2005, p. 129. (Russian)

state security. Policing on political grounds was also carried out by the organs of the NKVD, and they were empowered to impose sentences without a formal trial. A penal system was also part of the apparatus of the NKVD—the legendary GULag (State Labor Camp Administration). The practice of the NKVD at that time was such that when a “case” arose, the person accused of anti-soviet activity was, with 100-percent certainty, already in the grip of the GULag. The atmosphere in the country was one of intensified struggle with the enemies of Soviet power, and these enemies were sought out and found by the NKVD everywhere: from remote villages in Siberia to the leading party and governmental institutions. The underlying source of these trials was a struggle of various groups at the very highest levels of power. The trial of the “Trotsky-Zinov’ev Terrorist Center,” which took place in August 1936, in one count of which, it appears, certain highly placed Soviet functionaries wished to include the “case of Academician Luzin,” proceeded in accordance with what had become a standard template, a principal component of which was “condemnation by the whole people”—by the press, at meetings of workers, and so on. And although the time of greatest terror had not yet arrived, the gathering clouds of the massive repressions of 1937 could already be perceived. Luzin’s position was extremely grave. The transcripts of the meetings published here, meetings at which Luzin’s students openly persecuted him, leave a very dismal impression. The realization that these students were prominent mathematicians whose contributions to science are truly enormous, only strengthens that impression.

Fortunately for Luzin and for the whole Soviet mathematical school, the “case” ended on favorable terms: Stalin realized that a negative outcome would hinder rather than advance his policy of building the Academy, the General Headquarters of Soviet Science. If the investigation of this “case” had ended in the condemnation of Luzin as an “enemy of the Soviet state,” an outcome that was entirely possible—as the materials published in the present book show, that is what some of the organizers of the case from the Moscow and Central Committees of the All-Union Communist Party of Bolsheviks were counting on—the result would have been not merely the tragedy of a great mathematician, but would have had dire consequences for the mathematicians of Moscow and the entire country. That is exactly what happened with Soviet biology when the prominent geneticist N. I. Vavilov (1887–1943) was arrested and the “people’s” academician T. D. Lysenko (1898–1976) took the helm of Soviet biology. It is quite likely that the brilliant phenomenon of Soviet mathematics would never have happened, and that would have been a loss for all of world mathematics.

In the West, the “Luzin case” was perceived not simply as a dramatic episode affecting only Soviet or indeed only Moscow mathematicians, but as an event that affected the entire world community of mathematicians. Solidarity with Luzin was demonstrated by the mathematicians of France and Poland, who tried to find ways of bending the Soviet leadership in a direction favorable to the mathematical community. Subsequently, when a taboo was imposed in the Soviet Union on any mention of the affair, it was in the West, especially France, that the study of its history was begun by the late Pierre Dugac (1926–2000).

Personalities involved in the affair. A large number of mathematicians were involved in the investigation of the Luzin case, and further information about them will be found in the numerous remarks that accompany the documents published here. Here we confine ourselves to a list of the most prominent ones in

the order of their birth, along with their fields of research and most important achievements.

Krylov, Aleksei Nikolaevich (1863–1945). Elected a full member of the Russian Academy of Sciences in 1916, he had become a corresponding member in 1914. His main results are in shipbuilding, mechanical engineering, theory of gyroscopes, mathematical physics, approximation theory, and history of mathematics. (In 1915/16 he published a Russian translation of Newton's *Mathematical Principles of Natural Philosophy*.)

Bernshtein, Sergei Natanovich (1880–1968). Elected a full member of the USSR Academy of Sciences in 1929, he had become a corresponding member in 1924. In 1955 he was elected a foreign member of the French Academy of Sciences, of which he had been a corresponding member since 1928. His main results are in the theory of partial differential equations (in 1904 he solved the nineteenth of Hilbert's famous twenty-three problems), probability theory (in 1917 he gave the first axiomatization of the theory) and its limit theorems and the theory of weakly dependent random variables, and the applications of probability theory in physics and biology,

Golubev, Vladimir Vasil'evich (1884–1954). Elected a corresponding member of the USSR Academy of Sciences in 1934. His main results are in aero-mechanics and theory of functions of a complex variable and the analytic theory of differential equations.

Vinogradov, Ivan Matveevich (1891–1983). Elected a full member of the USSR Academy of Sciences in 1929 a foreign member of the London Royal Society in 1942, the Danish Royal Society in 1947, the Academy of the German Democratic Republic in 1950, the Hungarian Academy of Sciences in 1950, the Accademia dei Lincei of Rome in 1958, the Serbian Academy of Sciences in 1959, and the German Leopoldina Academy of Scientists in 1962. He was one of the greatest number theorists in the world. He developed the method of trigonometric sums, which became fundamental to the theory. He also contributed important results on Waring's Problem and solved the ternary Goldbach Conjecture.

Shmidt, Otto Yul'evich (1891–1956). Elected a full member of the USSR Academy of Sciences in 1935, he had been a corresponding member since 1933. From 1939 to 1942 he served as its vice-president. His main area of research was finite group theory, and he was the founder of the Moscow School of Algebra. He undertook the development of mathematical methods of studying the Kursk magnetic anomaly and proposed a cosmogonic conjecture on the evolution of the planets in the solar system. He was an active Arctic explorer.

Men'shov, Dmitrii Evgen'evich (1892–1988). Elected a corresponding member of the USSR Academy of Sciences in 1953, he was the author of fundamental results in the theory of trigonometric series. He discovered null-series, which are series that converge almost everywhere to zero, yet have non-zero coefficients, in 1916 and is co-discoverer of the Men'shov–Rademacher theorem on convergence of orthogonal series (1922–23); he is also known for what is called Men'shov's theorem on an integrable periodic function (1941) and other results.

Khinchin, Aleksandr Yakovlevich (1894–1959). Elected a corresponding member of the USSR Academy of Sciences in 1939. His main results are in the theory of functions of a real variable (the asymptotic derivative and the Denjoy–Khinchin

integral), number theory (the metric theory and the theory of Diophantine approximation). He was one of the founders of the modern theory of probability (stochastic processes, the iterated logarithm and others) and one of the founders (jointly with Kolmogorov) of the theory of random processes and of quality control and queueing theory. He was a leader in the reform of secondary mathematical education in the USSR from the mid-1930s to the 1940s.

Aleksandrov, Pavel Sergeevich (1896–1982) Elected a full member of the USSR Academy of Sciences in 1953, he had been a corresponding member since 1929. He was elected to the Göttingen Academy of Sciences in 1945, the National Academy of Sciences of the USA in 1947, and the German Leopoldina Academy of Scientists in 1959. From 1958 to 1962 he was vice-president of the International Mathematical Union. He was the founder (jointly with Uryson) of the Soviet School of Topology and one of the most prominent topologists of the twentieth century. He began his academic career in 1916 by proving the continuum conjecture for an uncountable Borel set (a result obtained simultaneously and independently by Felix Hausdorff). His main results are in topology: the Aleksandrov compactification, the Aleksandrov–Hausdorff theorem on the cardinality of A -sets, the Aleksandrov topology, Aleksandrov–Čech homology and cohomology, and more.

Lyusternik, Lazar' Aronovich (1899–1981). Elected a corresponding member of the USSR Academy of Sciences in 1946. His works encompass a very broad range of areas of mathematics: both ordinary and partial differential equations, topology, calculus of variations, functional analysis, geometry, mathematics of computation, special functions and much more. One of his most important achievements was the creation (jointly with Shnirel'man) of a new fundamental area of research: topological methods in analysis. Along those lines he solved the classical Poincaré three-geodesic problem, jointly with Shnirel'man.

Lavrent'ev, Mikhail Alekseevich (1900–1980). Elected a full member of the USSR Academy of Sciences in 1946, a foreign member of the Academy of Sciences of Czechoslovakia in 1963, the Bulgarian Academy of Sciences in 1966, the French Academy of Sciences in 1971. From 1957 to 1976 he was vice-president of the USSR Academy of Sciences. He was one of the organizers of the Siberian Division of the USSR Academy of Sciences and its President from 1957 to 1976. He was vice-president of the International Mathematical Union from 1966 to 1970. His research was in various areas of mathematics (primarily the theory of functions of a complex variable, calculus of variations, and mathematical physics) and mechanics (primarily solid state mechanics).

Bari, Nina Karlovna (1901–1961). Her main results are in real-variable theory, primarily the theory of trigonometric series.

Novikov, Pëtr Sergeevich (1901–1975). Elected a full member of the USSR Academy of Sciences in 1960, he had been a corresponding member since 1953. He was one of the founders of the Soviet School of Mathematical Logic. His main results are in the areas of set theory, mathematical logic and foundations of mathematics, theory of algorithms, and group theory. He obtained, jointly with his student S. I. Adyan, a negative solution of Burnside's periodic group problem.

Kolmogorov, Andrei Nikolaevich (1903–1987). One of the greatest mathematicians of the twentieth century, he was elected a full member of the USSR Academy

of Sciences in 1939, an honorary member of the American Academy of Arts and Sciences in 1959, a member of the German Leopoldina Academy of Scientists in 1959, a foreign member of the Royal Netherlands Academy of Sciences in 1963, a member of the London Royal Society in 1964, an honorary member of the Romanian Academy of Sciences and the Hungarian Academy of Sciences in 1965, a foreign member of the National Academy of Sciences of the USA in 1967, the French Academy of Sciences in 1968, the Academy of Sciences of the German Democratic Republic in 1977, an honorary member of the International Academy of History of Science in 1977, and a foreign member of the Finnish Academy of Sciences in 1985. He was one of the founders of modern probability theory and established an axiomatization of it. He was the author of outstanding results in topology, geometry, mathematical logic, classical mechanics, theory of turbulence, complexity of algorithms, information theory, theory of functions, trigonometric series, measure theory, approximation theory, set theory, differential equations, dynamical systems, functional analysis, statistical mechanics, and fundamental work in the history of mathematics.

Shnirel'man, Lev Genrikhovich (1905–1938). Elected a corresponding member of the USSR Academy of Sciences in 1933. Following Khinchin, he began to develop metric methods in number theory and introduced the concept of the density of a sequence in the series of natural numbers. This concept enabled him to obtain important results in the additive theory of numbers. One of his most important achievements was the founding (jointly with Lyusternik) of a new fundamental area, the area of topological methods in analysis. Along those lines, he solved the classical Poincaré three-geodesic problem, jointly with Lyusternik.

Gel'fond, Aleksandr Osipovich (1906–1968). Elected a corresponding member of the USSR Academy of Sciences in 1939 and a corresponding member of the International Academy of History of Science in 1963. His main results are in number theory and theory of functions of a complex variable. In 1934 he solved Hilbert's Seventh Problem, proving the transcendence of numbers of the form α^β , where α is an algebraic number different from 0 and 1, and β is an irrational algebraic number. He also studied the problems of cryptography and history of mathematics.

Pontryagin, Lev Semënovich (1908–1988). Elected a full member of the USSR Academy of Sciences in 1958, he had been a corresponding member since 1939. Elected an honorary member of the Hungarian Academy of Sciences in 1972. From 1970 to 1974 he was vice-president of the International Mathematical Union. One of the greatest mathematicians of the twentieth century, his main results are in topology, algebra, theory of oscillations, calculus of variations, optimal control, and game theory. He founded the mathematical theory of optimal processes, which is based on the Pontryagin Maximal Principle.

Sobolev, Sergei L'vovich (1908–1989). Elected a full member of the USSR Academy of Sciences in 1939, he had been a corresponding member since 1933. Elected a foreign member of the Accademia dei Lincei in Rome in 1966, a member of the Academy of Sciences of the German Democratic Republic in 1967 and the French Academy of Sciences, of which he had been a corresponding member since 1967, in 1978. One of the greatest mathematicians of the twentieth century. His research was in partial differential equations, calculus of variations, integral equations, functional analysis, approximate and numerical methods, and mathematical analysis.

He pioneered the application of functional analysis to partial differential equations. He introduced the concept of generalized functions in 1935/36 and developed the theory of a number of problems involving linear partial differential equations in terms of generalized functions. He defined a class of function spaces (Sobolev spaces) and studied embedding relations for these spaces (the Sobolev Embedding Theorems). In the 1940s and 1950s he participated in work on the “atomic project.” He was one of the pioneers in the USSR in the field of computational mathematics and application of computers. In the 1960s he developed the theory of cubature formulas.

It goes without saying that the descriptions given above are incomplete. The interested reader will find a great deal more in the present book and also in the extensive literature found, for example, in the bibliography to the book mentioned in Footnote 13 below. But even these scant data make it possible to appreciate the level of mathematicians involved in the “case,” which became an important turning point in the history of the mathematics of the twentieth century. All of them were among the elite in the mathematics of the century. Among the prominent figures in the mathematical Moscow of the time not on this list we mention B. N. Deloné and N. E. Kochin, who had only recently come to Moscow and therefore had the formal right not to participate actively in the investigation, and also the Moscow mathematicians more or less connected with Luzin, but who also declined to join the ranks of the attackers, such as Keldysh, Petrovskii, Privalov, Stepanov, and Tikhonov. Nowhere in the materials of the affair do we encounter the name of one of the greatest mathematicians of the century Izrail Moiseevich Gel’fand (1913–2009), who providentially was in no way connected with Luzin or with the circumstances of the “case”. He had arrived in Moscow only in 1930 and it was not until 1932 that he entered Moscow University, where in 1935 he defended his *kandidat* dissertation, a dissertation that became, as V. M. Tikhomirov¹² writes, a classic work in functional analysis.

The circumstances of the “case” left a deep wound on the body of the Soviet mathematical community, one that was not to heal for many years. These circumstances caused a rift among Luzin’s immediate students, who were, as we have seen, prominent Soviet mathematicians, dividing those who participated in the persecution of their teacher from those who refused to join them. This estrangement also affected the next generation of students of Luzin’s students. It is very important to emphasize that this “case” served as a serious lesson for the Soviet mathematical community, a lesson well learned by its leaders. They realized what danger might come to the community might from scientific ideologues such as E. Kol’man (1892–1979), one of the organizers of the “Luzin case,” and they did everything possible to keep such people away from the leadership of the community. At the same time, the circumstances of the “case” and the whole experience of the 1920s and 1930s had shown the impossibility of living in isolation from ideology in the midst of a thoroughly ideologized society. One could not merely say, “We are studying pure science and ideological problems don’t exist for us.” They realized that the official ideology had to be accepted, at least externally (those were the rules of the game!) and what is more, actively. Only under such conditions would it be possible to study

¹²Tikhomirov, V. M., “Izrail Moiseevich Gel’fand,” *Mathematical Education*, Third series, No. 8 (2004), pp. 8–12 (Russian).

science in peace. That is why they themselves elected as official ideologues people whom they did not regard as dangerous to themselves, with whom they could deal; such a person was S. A. Yanovskaya (1896–1966). Moreover, they themselves began to participate in the discussion of questions involving a world-view, furnishing rational foundations for this work and avoiding dangerous ideological extremes. An example of such an experiment was the 1956 publication of the three-volume *Mathematics, its Content, Methods, and Meaning* under the editorship of Aleksandrov, Kolmogorov, and Lavrent'ev, written by the leading mathematicians of the country. This established the necessary external conditions for the normal functioning of the mathematical community and the consequent flowering of the Soviet mathematical school in the 1960s and 1970s.¹³

¹³ There is an extensive literature on the Soviet school. See, for example, S. Zdravkovska and P. L. Duren (eds.), *The Golden Years of Moscow Mathematics*, American Mathematical Society: History of Mathematics, Vol. 6, Providence, RI, 1991, and also numerous articles in the second series of *Istoriko-Matematicheskie Issledovaniya*, published by Yanus-K, Moscow. Nos. 1 (36)–15 (50), 1995–2014.

Enemies Wearing a Soviet Mask (*) (**)

Pravda, 3 July 1936, p. 2

The letter of Comrade Shulyapin, director of School No. 16, published yesterday in *Pravda* ("Response to Academician Luzin") in reply to Academician Luzin's article in *Izvestiya*, has lifted the fringes of the curtain under which Academician Luzin's "activity" takes place. On the very day it was published, this letter evoked a number of sympathetic responses and letters to the editors of *Pravda* from people working in the mathematical sciences. A closer inspection of the activity of this academician over the last few years shows that the artful raptures lavished on our pupils by Luzin are by no means accidental. They constitute merely one link in a long chain of skillful and, in their methods, very instructive, masquerades of an enemy.

Most remarkable of all is the generosity with which Academician Luzin heaps laudatory evaluations on certain claimants to the title of professor or doctor of science. For example, regarding the book *A Brief Course of Higher Mathematics* by A. K. Uspenskii, Luzin wrote to the Higher Attestation Commission that this textbook "reveals a perfect applicability to instruction" and that "an examination and analysis of these works shows the author's highly developed mathematical culture and attests to his right to occupy a chair." But when the book was conscientiously examined by other professors, it turned out, as stated by the Higher Attestation Commission, that ***A Brief Course of Higher Mathematics contains many very crude errors and cannot be regarded as a textbook. . . It is replete with turns of phrase so mathematically illiterate that they are unacceptable even for use with good students.***

Luzin has given still more sparkling and equally undeserved evaluations on the works of V. Eiges, V. Deputatov, P. Bessonov, and many others. Thus, inexcusably praising Eiges, he wrote:

"Eiges is the author of quite profound, valuable, and interesting research in the foundations of geometry. This research is the more important in that it is completely **original**, designed and carried out by the author **independently** as a lone investigator without any external influences."

An examination of this work by Prof. Khinchin revealed that "both the present report and several other manuscripts presented by the author devoted to various problems of geometry **are completely on the level of expository work and do not contain even minimally serious scholarly research.**

But perhaps these are merely errors, the scholarly errors of an honest scholar? No. Academician Luzin himself, in conversations with his friends, laughs at his evaluations and believes that, for example, one could not award any academic degree to either Deputatov or Bessonov. The fabrication of demonstrably false laudatory

evaluations is one point on the line of Academician Luzin, a line aimed at cluttering up Soviet mathematical science with underqualified people. Academician Luzin has been working systematically along this line for several years now, right up to the last few months and days, recommending for work at the Academy of Sciences people who at best need to take a course of study at a university. Such was the case with K. Ivanov, very recently with Sh. Shadkhan, and others, as members of the Mathematical Group of the Academy of Sciences write in *Pravda*.

This egregious disruption of scholarship by Academician Luzin is readily apparent and follows the example of his own papers. In order to appear to be an active member of the Academy of Sciences he publishes many of his allegedly scientific papers in USSR publications. But the academic value of the majority of these articles is insignificant. In his conversations with friends, Luzin himself does not hesitate to say that he dumps several memoirs a year onto the scientific market while inwardly laughing at their content. He calls these papers rubbish that one has to publish only to “intimidate by quantity.” These—pardon the expression—“works” are deliberately padded, and F. R. Gantmakher, a graduate student at the Academy of Sciences, proved at the Second All-Union Mathematical Congress that the results of a series of papers by Luzin on the method of Krylov (formation of the secular equation), occupying 160 pages, could easily be accommodated in . . . three pages. Luzin publishes obituaries, informal and insubstantial notes, commentaries, and the like, as scholarly papers. His more substantive works he sends abroad for publication, to France, Poland, and even . . . Romania.

On the other hand, the independence of many of Luzin’s papers is subject to doubt, for he does not hesitate to present the discoveries of his students as his own work. Thus, going back to 1917, the young scholar M. Suslin discovered a new class of sets of fundamental importance, the so-called *A-sets*. Luzin did everything in his power to get his student Suslin out of Moscow and make it impossible for him to work. And no sooner did Suslin die (in 1919) than Luzin hastened to take advantage of that in the works that he published abroad, presenting the discovery of the student he had persecuted as his own. Even now, Luzin does not hesitate to publish the work of his students under his name, as happened last year in the book *On Some New Results in Descriptive Function Theory*. The cover of this book is adorned by the name of Luzin, while the work inside is that of his student P. Novikov.

Combining this moral depravity and scientific perfidy with a concealed hostility and hatred to everything Soviet, Luzin has adopted the tactic of “being as wise as a serpent.” He supposes that the Soviet citizens around him are fools, who can be misled and deceived endlessly and without hindrance while he hides behind the august title of Soviet scholar. But this semirespected Academician forgets that Bolsheviks are very good at recognizing serpents, no matter what hide they dress up in. We know very well that Luzin is an anti-Soviet man.

Academician Luzin plays on flattery (sometimes very subtle flattery) of Soviet scientific youth—letting on that it already knows everything, is a mature scholar, and so on and so on. But he secretly sneers and bares his fangs when he talks privately with his friends, saying that none of this is serious and there is hope that the time of youth is coming to an end, and he blackballs truly talented young scholars, keeping them out of the Academy.

Soviet mathematical science has great virtues and great achievements. Mathematics is flourishing in the Soviet land, like all the other sciences for which the Socialist government of workers and peasants has provided conditions that do not exist and cannot exist in any capitalist country. We know where Academician Luzin grew to maturity, that he is a scion of the infamous tsarist "Moscow Mathematical School," whose philosophy was that of the Black Hundreds and whose motivating ideals were the leviathans of Russian reaction: Orthodoxy and Autocracy. We know that even today he is not far from such views, perhaps fascistically updated to a small degree. But the social soil that grew Luzin is a thing of the past; it has been vanishing and continues to vanish from under our feet. Academician Luzin could have become an honorable Soviet scholar, as many of the older generation did. He was unwilling to do so. He, Luzin, remained an enemy, relying on the power of social mimicry, and the impenetrability of the mask he had crammed onto his face.

It won't work, Mister Luzin! The Soviet scholarly community has torn off your mask of a conscientious scholar, and you now stand before the world naked and insignificant. You, preaching "pure science" and all the while betraying the interests of science, selling it out in the interests of your former bosses and the present bosses of fascist science. The Soviet community will perceive the history of Academician Luzin as yet another object lesson showing that the enemy is not laying down his arms, that he is masquerading in ever more skillful ways, that his methods of mimicry become more and more varied, and that vigilance remains the most important trait of every Bolshevik and every Soviet citizen.