

Preface

Max Dehn's name is known to mathematicians today mostly as an adjective (Dehn surgery, Dehn invariant, etc.). He is also remembered as the first mathematician to solve one of Hilbert's famous problems (the third) as well as for pioneering work in combinatorial topology. Beyond his accomplishments as an eminent mathematician, however, he was a remarkable scholar and teacher whose influence extended far beyond research mathematics. He also led a remarkable life, parts of which are described in this book along with some of his most important contributions to mathematics and their afterlife. This book's three-part structure is based on three distinct periods in Dehn's life. The first centers on his early career as one of David Hilbert's most gifted students. During the second phase following the First World War, he emerged as the leader of an intimate community of mathematicians in Frankfurt, an idyllic era that quickly darkened once the Nazi government seized power in 1933. After fleeing to Scandinavia in 1938, the Dehns had to undertake an arduous journey that brought them to the United States. The third part takes up his initial wanderings in the US before focusing on his years at Black Mountain College, where he found a final home.

As the title of this book suggests, many voices contributed to the chapters. The central events in Dehn's life are presented in chapters 1, 2, 5, and 9 through 12. These are interspersed with four other chapters (3, 4, 7, and 8) that focus on mathematical themes that are found in Dehn's work and his influence on their development. Chapter 6 presents a mixture of biography and mathematical exposition in a discussion of the work of Dehn's three most prominent students. A great deal of the information found in all twelve chapters has never been presented before. The various perspectives and interests are often original as well. The book, however, remains a collection of essays; it does not pretend to offer a comprehensive picture – a full-scale scientific biography of Max Dehn remains to be written. That being so, we nevertheless feel confident that this book contributes significantly to the literature and readers of all sorts will find something new and enriching.

Max Dehn was born in Hamburg, Germany, on 13 November 1878, the fourth of eight children of Maximilian Dehn, a medical doctor, and his wife Berta, née Raf. The family belonged to Hamburg's vibrant middle-class Jewish community, which was mainly comprised of secular rather than religious Jews. Dehn's family background is described in Chapter 1.¹ After spending a single semester in Freiburg, he entered Göttingen University in the summer of 1897 (Chapter 2). It took him altogether only six semesters to complete his studies, so he had not yet reached his twenty-first birthday when in November 1899 he passed his doctoral examination

¹Further information about the eight children and their relatives can be found in Matthias Brandis, *Meines Großvaters Geige. Das Schicksal der Hamburger jüdischen Familien Wohlwill und Dehn*, Leipzig: Hentrich & Hentrich.

and took his degree *summa cum laude*. Soon thereafter, his star was already clearly visible in the mathematical firmament, illuminated by his noteworthy contributions to the foundations of geometry (Chapters 2 and 3).



FIGURE 1. Max Dehn, second from right in top row, with the Göttingen Mathematical Society, 1899. To Dehn's left stands Harvard's William Fogg Osgood, a former student of Felix Klein (seated in the front row, third from left, next to David Hilbert, second from left). In front of Max Dehn, to his right, is Alexander Ziwet from the University of Michigan. Ziwet, a native of Breslau, is occasionally remembered for having performed the service of preparing the written text based on Klein's *Evanston Colloquium Lectures* from 1893.

During those years, American and other foreign mathematicians flocked to Göttingen to study or visit, if only during the summertime, since the universities in Germany normally remained in session until early August. The photo of the young, innocent-looking Max Dehn (Figure 1) standing next to much older mathematicians was probably taken in the summer of 1899, possibly in June during the celebrations at the time of the unveiling of the Gauss-Weber Monument. That ceremony had special significance for Dehn, since he had been intensely following Hilbert's lectures on the "Elements of Euclidean Geometry" during the previous winter semester. Thus, no one was in a better position to appreciate the importance of Hilbert's contribution to the *Gauss-Weber Festschrift*, the original version of his *Grundlagen*

der Geometrie.² In later years, Dehn would help his former mentor refine and improve that original edition.

The mathematician Francisco José Craveiro de Carvalho described the young man in that photograph poetically:

This is the summer season
of the quiet year of 1899
in Göttingen.
In the background there is a river
or a stage-setting that creates a river.

Only we still know
that the young Max Dehn
will have to rush
out of the photograph
and get rid of his coat
high-collared shirt and former life
to try finding a new one for himself
on a strange and distant shore.

(Translated from the Portuguese by Manuel Portela)³

Max Dehn's years as a student seemed to set him off on a fast track. Yet, as the biographical chapters in our portrait show, his academic career was a long uphill climb, not without setbacks along the way (Chapters 2 and 5). Indeed, seen from a longer term perspective, he faced many of the same obstacles as other young men from this era. The religious confession of candidates was at times a definite factor, though ethnicity rather than religious affiliation played an even larger role during the first phase of his career. After spending more than a decade as a private lecturer (*Privatdozent*) in Münster, Dehn received his first appointment as an associate professor in Kiel, where his candidacy for a post as full professor was passed over. Shortly thereafter, however, he gained that promotion in 1913 with an appointment to the Technical University in Breslau, where his work was interrupted by three years of war service.

Dehn made his major contributions to mathematics roughly between 1900 and 1920. He was attracted by the newest currents of research, beginning with his thesis work on the foundations of geometry. Riding on the wave of interest generated by Hilbert's work, he became a leading authority in this field (Chapters 2 and 3). His interest in the geometry of polyhedra led to his solution of Hilbert's third Paris problem, and his attempt to solve the Poincaré conjecture took him into the vast realm of 3-dimensional manifolds in topology (Chapters 3 and 4). In all this work, Dehn fashioned powerful tools that took on lives of their own. For example, his invariant for polyhedra involved a geometric property of rectangles filled by sub-rectangles – a subsequent paper of Dehn's has been recast and reproved

²There are significant differences between this text and the many editions that followed it. A reprint of the *Urtext* along with commentary on it and subsequent editions can be found in Klaus Volkert, Hrsg., *David Hilbert, Grundlagen der Geometrie (Festschrift 1899)*, Heidelberg: Springer, 2015.

³The original Portuguese version of this poem was first published in *Gazeta de Matematica*, # 149, July 2005.

by many geometers since its publication (Chapter 3). His work on 3-manifolds required a deeper understanding of the fundamental groups of surfaces, for which he developed what became the foundation of geometric group theory, a thriving field today (Chapter 7). Dehn's pioneering studies in combinatorial group theory also opened the door to research on the word problem, as described in Chapter 8.

After the war, Dehn returned to Breslau, but his career entered a new phase in 1921 with his appointment in Frankfurt, where he soon became head of its mathematics program (Chapter 5). Today this is remembered as a "golden age" for mathematics in Frankfurt, whose faculty was unusually harmonious and dedicated. Dehn's former Göttingen teacher, Arthur Schoenflies, headed the original faculty, acting as Dean in 1920/21. Max Dehn's appointment not only elevated his own career, it also led to the appointments of Hungarian Otto Szász and Paul Epstein as associate professors in 1921. At the outset of the new republic, the Prussian Ministry supported the promotion of scholars (including many Jews) who had been held back under the monarchy. Schoenflies retired one year later, opening the way for young Carl Ludwig Siegel to assume his position. A key figure during the 1920s and early 30s was Ernst Hellinger, whom Hilbert had chosen earlier as his assistant. Hellinger came to Frankfurt University at the time of its founding in 1914, and during the Dehn era he took charge in organizing the course work and advising students.

Siegel would later memorialize this unique community in a moving lecture, delivered to members of the new faculty in 1966.⁴ He warmly remembered the atmosphere created by these selfless colleagues, but especially their weekly historical seminar, led by Dehn, in which they read classic mathematical texts in their original languages (Chapter 5). He also described how this brilliant era came to an abrupt end once the Nazis came to power in 1933. The Dehn children thereafter found refuge in England and the United States, leaving their parents behind in Frankfurt, where Dehn's position was terminated in 1935. Then came the terrifying pogroms during the "Night of Crystal Glass" in November 1938, after which Dehn and his wife Toni fled to Scandinavia (Chapter 9).

They first went to Copenhagen in January 1939 before moving on to Trondheim in Norway, where Dehn took over the post of a vacationing colleague at the Technical University. With the German invasion in 1940, however, hopes of remaining in Norway suddenly vanished. Thanks to support from friends in the United States, the Dehns were able to leave Oslo in late October 1940. Chapter 10 describes their long journey across Siberia, reaching San Francisco at the very beginning of 1941, followed by four years of temporary teaching jobs across the country, a story recounted in Chapters 10 and 11. Max Dehn was remembered by many of his students in the US as a man of unusually broad interests.

Dehn finally found refuge, and a setting for his many talents, at Black Mountain College (BMC) in North Carolina (Chapter 12), a unique though short-lived experiment in higher education situated in the mountains of western North Carolina. During the last eight years of his life (1945–1952), he taught mathematics, philosophy, Greek, and Italian at BMC. The curriculum at Black Mountain College focused on arts and crafts, and BMC is celebrated today for having served as a

⁴C.L. Siegel, On the History of the Frankfurt Mathematics Seminar, *Mathematical Intelligencer* 1(4): 223–230. Wilhelm Magnus, Max Dehn, *The Mathematical Intelligencer*, 1(3), 132–143, 1978.

catalyst for several renowned artists. In addition to teaching elementary mathematics and projective geometry and tutoring a few advanced mathematics students at BMC, Dehn also taught a very popular course called “Geometry for Artists,” while working in close association with others on the BMC faculty. Love of nature had always been a central part of his life, and he felt especially at home in these surroundings, far from what others would call civilization.

When he died, “Dehn” was not yet the adjective familiar to mathematicians today. As his former student Wilhelm Magnus remarked at the Max Dehn centennial in 1978, “his name is probably more widely known and certainly more widely-used today than it was at the time of his death in 1952.” In present-day nomenclature we find the Dehn invariant, Dehn twists, Dehn’s Lemma (about which, see Chapter 4), Dehn’s algorithm, Dehn filling, and Dehn surgery. Terms that carry names honor (sometimes incorrectly) the work of the mathematician who introduced them. Dehn’s influence can clearly be seen in the work of three of his most distinguished students (see Chapter 6): Jakob Nielsen worked on the topology of surfaces, Wilhelm Magnus studied groups through geometry and topology, and Ruth Moufang took up the foundations of geometry in the Hilbertian manner by building bridges between geometry and algebra. Thus, while this book focuses on Dehn’s life, the temporal range extends to the end of the twentieth century through the afterlife of his works and the research of his students.

To provide an overview, we here list various milestones in Max Dehn’s career, framed by the structure of the book.

Part 1: From Hamburg to Breslau (1878–1921)

- 1878–1896 Early years in Hamburg
- 1896–1900 Student in Freiburg and Göttingen
- 1900 PhD Göttingen under David Hilbert (Legendre’s Theorems on the Sum of the Angles in a Triangle, published in *Mathematische Annalen* 53(1900): 404–439)
- 1900/01 Assistant of Friedrich Schur at the Technische Hochschule Karlsruhe
- 1901 Habilitation in Münster under Wilhelm Killing (*Habilitationschrift* on Hilbert’s Third Paris Problem, “Über den Rauminhalt”, published in *Mathematische Annalen* 55(1902): 465–478)
- 1901–1911 Privatdozent in Münster
- 1911–1913 Extraordinarius in Kiel
- 1913–1921 Ordinarius at the Technische Hochschule Breslau
- 1915–1918 Service in World War I on the Western Front

Part 2: Frankfurt Years (1921–1938)

- 1921–1935 Ordinarius in Frankfurt/Main
- 1922 Founding of the Frankfurt Mathematics Seminar
- 1935 Dismissed on formal grounds immediately prior to implementation of the Nazi racial laws (he receives regular pension payments until 1940)
- 1938/January to April teaches at the boarding school attended by Dehn’s children in Kent/England

Part 3: From Frankfurt to the USA (1939–1952)

- 1939 Emigration (Copenhagen and Oslo)
- 1939/40 Visiting professor in Trondheim/Norway (substituting for Viggo Brun)
- 1940/October 30 The Dehns leave Oslo for San Francisco.
- 1941/January Arrival in USA (via Sweden, Finland, Soviet Union, Japan)
- 1941/42 Asst. Prof. of mathematics and philosophy, University of Idaho–Southern Branch (today Idaho State University)
- 1942/43 Visiting Prof. of mathematics, Illinois Institute of Technology, Chicago
- 1943/44 Tutor, St. John’s College, Annapolis, Maryland
- 1945–1952 Prof. of mathematics and philosophy, Black Mountain College
Lecturing at the University of Wisconsin, Madison 1946/47 (fall semester),
1948/49 (fall and spring semesters), and Notre Dame 1949 (summer)
- 1952 Prof. em., Black Mountain College

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The editors