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## PREFACE

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Game theory sheds a light on many aspects of the social sciences and is based on an elegant and non-trivial mathematical theory. The bestowal of the 1994 Nobel Prize in economics upon the mathematician John Nash underscores the important role this theory has played in the intellectual life of the twentieth century. There are many textbooks on this topic but they tend to be one sided in their approaches. Some focus on the applications and gloss over the mathematical explanations while others explain the mathematics at a level that makes them inaccessible to most non-mathematicians. This monograph fits in between these two alternatives. Many examples are discussed and completely solved with tools that require no more than high school algebra. These tools turn out to be strong enough to provide proofs of both von Neumann's Minimax Theorem and the existence of the Nash Equilibrium in the  $2 \times 2$  case. The reader therefore gains both a sense of the range of applications and a better understanding of the theoretical framework of two deep mathematical concepts.

This book is based on lectures I presented in **MATH 105 Introduction to Topics in Mathematics** as well as in **MATH 530 Mathematical Models I** at the University of Kansas. The first of these courses is normally taken by Liberal Arts majors to satisfy their Natural Sciences and Mathematics Distribution Requirements. The presentation of Chapters 1-9 and 11-13 in this class took about 25 lectures and was supplemented with notes on statistics, linear programming and/or symmetry. In the mathematical models class this material was used to supplement a standard linear programming course. It can be covered in about a dozen lectures with proofs included in both the presentation and the homework assignments. Those chapters and exercises that are deemed to be more theoretically demanding are starred. Such proofs as are included in the text appear in the conclusion of the appropriate chapters.

The exposition is *gentle* because it requires only some knowledge of coordinate geometry, and linear programming is *not* used. It is *mathematical* because it

is more concerned with the mathematical solution of games than with their applications. Nevertheless, I have included as many convincing applications as I could find.

I am indebted to my colleagues James Fred McClendon for helping me out with some of the technical aspects of the material and Margaret Bayer for rooting out some of the errors in an earlier draft. I owe my own understanding of the material and many examples to the books by John D. Williams and David Gale. David Bitters, Sergei Gelfand, Edward Dunne, and an anonymous reviewer contributed many valuable suggestions. Larisa Martin and Sandra Reed converted the manuscript to TeX and Sandra Donnelly supervised the production. To them all I owe a debt of gratitude.

Saul Stahl