Preface

This volume is the first in a series of translations of selected mathematical articles published in the Russian magazine "Kvant" (meaning "Quantum") since 1970. American Mathematical Society plans to publish a number of volumes a year for several years to come. The papers are subdivided into three categories: algebra and analysis, geometry and topology, discrete mathematics (the present volume belongs to the former).

Most probably, the reader has never heard of "Kvant". The influence of this magazine on the mathematics and physics education in the Soviet Union is hard to overestimate. What follows is a brief history of "Kvant" and a modest tribute to the people who created it. This is also a personal account of my "love affair" with "Kvant" that spanned more than 20 years.

I was a seventh-grader when, 28 years ago, I opened the very first issue of "Kvant" (subscribed to by my parents). I could hardly understand a word: the material appeared way too difficult for me. I do remember my frustration (so familiar to a college student who is taking his first serious mathematical class) combined with the desire to explore and master the magnificent world of mathematics and physics about to open for me. I remained a devoted "Kvant" reader ever since. My involvement continued in the eighties as an author of about 20 articles published in the magazine. In 1988 I was offered the position of the head of the Mathematics Department at "Kvant", and I did not have to think twice to accept it (I occupied this position for two years and then left to teach at an American university).

"Kvant" was initiated in 1970 by a group of prominent Soviet physicists and mathematicians; its first editor-in-chief was a celebrated physicist Isaak Kikoin and the deputy editor-in-chief was a great mathematician Andrei Kolmogorov. Although the seventies were rather "dark" years in the history of Russia, "Kvant" was a product of the post-Stalin "thaw", and the spirit of independence was ever present there.

Physics and mathematics were extremely popular in the Soviet Union at the time (in part, because these disciplines were as distant from ideology as possible); special schools for physics and mathematics flourished, mathematical and physical olympiads attracted numerous high school students, and the entrance exams to departments of physics and mathematics of good universities were very hard and competitive (in Russia entrance examinations are specialized: for instance, a mathematics major to be has to pass four exams: written and oral mathematics, oral physics and composition).

"Kvant" was distributed by subscription only. In the seventies it had an incredible number of more than 300,000 subscribers, in the eighties this number slowly decreased to about 200,000, and in the nineties, with the collapse of the Soviet Union and drastic changes in all aspects of life in Russia, the circulation of the

magazine decreased dramatically. Since 1991 the English language sister-magazine of "Kvant", called "Quantum", has been available in the West; it is currently being published by Springer-Verlag ("Quantum" is not a cover-to-cover translation of "Kvant": some of the published materials previously appeared in "Kvant" and some are original contributions).

"Kvant" was a monthly magazine, and the subscription was very affordable (the price of an issue was, roughly, that of a lunch). The magazine was published by the two All-Union Academies: Academy of Sciences and Academy of Pedagogical Sciences; both were huge and rich organizations, and the resources of "Kvant" were substantial. In particular, all illustrations—and they were plentiful!—were in color. Along with the Editorial Board and the Editorial Counsel, the "Kvant" staff included about a dozen full-time editors (some of them with Ph.D), secretaries, typists (it was the pre-computer era). Unlike many popular magazines, all submitted articles had been refereed; the authors were paid a relatively generous honorarium.

A few words about a somewhat unconventional treatment of the submitted articles in "Kvant". My first encounter with this system goes back to the late seventies when, a recent college graduate and an aspiring "Kvant" author, I submitted my first article to the magazine; later, an insider, I learned more about working with "Kvant" authors as an editor. "Kvant" had very high standards of exposition, and most of the submitted materials did not satisfy these criteria. The acceptance of an article by the Editorial Board was, in many cases, only the beginning of a serious work on it. That included heavy editing, often complete revision and rewriting, sometimes jointly with the editors, and in some cases, compilation of several articles into one (it is impossible to imagine this kind of editing in, say, "American Mathematical Monthly"—just think about the legal issues involved!) The final product was often dramatically different from but invariably better than the original submission.

Who were the "Kvant" readers? Most of them were high school students and teachers; a significant part were mostly interested in the preparation materials for the university entrance examinations. Many school libraries subscribed to "Kvant", and it often happened that a single issue had more than one reader. Part of the readership consisted of undergraduate and graduate students, researchers and general public interested in physics and mathematics (on the negative side: "Kvant" have been receiving numerous trisection constructions, proofs of the Last Fermat Theorem, etc).

A great part of "Kvant" appeal came from its visual image. What I mean are not the technical figures illustrating the articles (they are reproduced in the present edition in black and white), but the pictures accompanying them (unfortunately not reproduced here); each issue contained a few, most of them full page sized (one can find similar pictures in "Quantum"). A number of award-winning book illustrators worked with "Kvant", which provided an opportunity to express themselves freely: their art was way too radical for the prevailing official taste. An additional challenge was to relate art with the mathematical and physical content. some artists requested a detailed explanation of the article to be illustrated. It was especially in the late eighties (the Head of the Art Department was Sergei Ivanov) that "Kvant" was providing world-class samples of book graphics. I hope that one day an exhibition of the "Kvant" art will be organized at an Annual Meeting of the American Mathematical Society.

Sometimes "Kvant" art had a personal touch too. Here is an example. You will find the article by V. Ovsienko "On the Denogardus great number and Hooke's law" in the next volume of this series. This article, whose topic is the Sturm theory of second order differential equation, is written as a mystification: its main character is a medieval scholar Denogardus, whose work of life was burned by the Inquisition (fortunately the very thickness of the manuscript prevented it from perish). The article is illustrated by two pages from Denogardus' book (with burned out margins); one is a portrait of the author involved in astronomical measurments. A close look reveals that Denogardus has suspiciously many features in common with V. Arnold, a celebrated Russian mathematician and a "Kvant" author.

A typical "Kvant" issue (64, and later, 80 pages long) contained four feature articles, two in physics and two in mathematics; there would be also one or two shorter articles under the headings "Physics Lab" or "Mathematical Circle". Every issue had a Problem Corner (hereafter referred to as *Kvant Problem Book*): five mathematical and five physical problems; their solutions would be published a few months later. Since the eighties, "Kvant" publishes articles and problems on computer science. A significant space was occupied by preparation materials for the entrance examinations to universities; this included articles on high school curriculum topics and problems from last year entrance exams at universities from all over the country. A magazine within a magazine, called "Kvant for elementary schoolers" (take the name with a grain of salt: the material was often far from elementary!), consisted of a short article and a Problem Corner; here are two samples:

The population of the Soviet Union is, roughly, 250 *million. Explain why there is not enough room for* 250 *people on the* $1 : 1,000,000$ *scale map of the Soviet Union.*

Throw a table tennis ball directly upward. Does the ball spend more time going up or down?

There were many smaller sections in the magazine: mathematical games, chess page, letters from the readers, book reviews, etc. Each "Kvant" issue contained 50–100 mathematical problems of various levels of difficulty, with or without solutions. From time to time interviews with prominent mathematicians and physicists appeared in "Kvant", for example, with V. Arnold, I. Gelfand, A. Kolmogorov, S. Novikov, R. Graham.

"Kvant" also published a great deal of information about summer schools, schools by correspondence, olympiads and contests, regional, national and international. Once a material was published about Fields medalists, with their brief biographies and pictures. At the end of the article the hope was expressed that one day a "Kvant" reader and author would become a Fields medalist; this hope materialized in 1998 when M. Kontsevich was awarded the Medal.

The present collection consists of translations of selected mathematical feature articles. These articles cover a great variety of topics, but they were always conceptual, nontechnical and provided a substantial introduction to the subject (sounds like a description of a good colloquium talk, doesn't it?). The authors did not feel obliged to stick to any particular curriculum (assuming only familiarity with the standard high school mathematics and physics; however, one routinely referred to other "Kvant" articles), and this often allowed them to cover much more material than in a systematic course, leaving technical details aside.

Many of the feature articles concerned classical mathematical gems, some of which are or used to be part of the university curriculum: Pascal's and Brianchon's

theorems in projective geometry; evolutes and involutes of plane curves; distribution of prime numbers; geometry of developable surfaces; possible and impossible ruler and compass constructions; arithmetic of cubic curves; Newton's proof of "no gravity in a cavity;" Chebyshev polynomials; Gauss–Bonnet theorem—to mention just a few. Classical results were connected, when possible, with contemporary mathematics, say, Euler's theory of developable surfaces with modern singularity theory (in the article by D. Fuchs, "Paper sheet geometry").

Other articles concerned the 20th century mathematics, sometimes cutting edge research. Here are some examples: Dehn's invariant and Hilbert's third problem; flexible polyhedra; commuting polynomials and rational functions; geometry and dynamics of mathematical billiards; the Jones polynomial of knots and links; Euler's, Gauss' and MacDonald's identities; quasicrystals; lattice points in polyhedra. Some of the articles contained original mathematical results. Many feature articles were biographical and historical; one was devoted to meters in Russian poetry and another to mathematical themes in M. Escher's art.

How much of all that did an average reader comprehend and digest? Probably not all of it, at least, not from the first reading—but one was supposed to return to difficult papers more than once. However, I believe that the readers have learned the following lessons: mathematics is united, not fragmented into a numerous unrelated pieces; mathematics is beautiful; fundamental mathematical concepts are simple; an instructive example is worth an abstract general theory; and a figure is worth a thousand words. One more lesson to learn was that one did not have to wait until graduate school to start mathematical research; in fact, one could start even in high school (and indeed, Russian students start independent research considerably earlier than in the West).

A few words about "Kvant" authors. Some of them are research mathematicians or physicists, and some are professional educators; occasionally an author is a college student or a layman. Many celebrated mathematicians regularly contributed to "Kvant". Here are some familiar names: A. Aleksandrov, P. Aleksandrov, V. Arnold, I. Bernstein, V. Boltyanskii, A. Fomenko, D. Fuchs, S. Gindikin, A. Katok, A. Kirillov, A. Kolmogorov, M. Krein, Yu. Matiyasevich, L. Pontryagin, V. Tikhomirov, N. Vilenkin (not too many mathematicians could boast having an audience of a hundred thousand!).

Finally, I would like to pay tribute to the person who, for me, impersonates "Kvant" more than anyone else. This is Nikolai Vasil'ev, who untimely passed away in 1998 (the reader of this collection will encounter this name more than once). He was with "Kvant" from the very first day of its existence. An accomplished mathematician, he probably published more in "Kvant" than any other single author. He was a member of the Editorial Board, and since 1970 he was in charge of *Kvant Problem Book*. Due to his excellent taste and broad mathematical erudition this Problem Book became one of the best collections of mathematical problems ever (comparable to that of "American Mathematical Monthly"; I hope that selected mathematical problems from "Kvant" with their solutions will be eventually included in this series of translations). Without people like N. Vasil'ev "Kvant" would not have happened.

Serge Tabachnikov