Preface

About two years ago, we developed a series of lectures on mathematics for high school students, which were held every other week at Kyoto University. Kenji Ueno, who is one of the authors of this book, began this program and was primarily responsible for initiating these lectures. This unique lecture format took root and grew due to Ueno's keen understanding of the education and learning process and the support and enthusiasm of approximately fifty high school and junior high students. Today you can find this mathematical lecture format not only at Kyoto University, but at many other universities in Japan.

On December 23 and 24, 1994, Kyoto University held a special program of lectures on Geometry as part of this series. The three authors of this book gave their presentations on different themes. After the scheduled lectures, two of the high school students who had attended asked if we have any plans to publish a book about the material we had just covered.

As it turns out, we were thinking about publishing a book based on this material because we felt that the lecture format could not reach enough people. Once we limit the presentation strictly to lectures, the audience drastically narrowed. Also, geographical limitations further reduce the number of people who have access to the information. So when we were asked if we would publish, we were determined to do so. At that time, though, we were not yet prepared to arrange and publish the lectures.

One night, one of the students suggested "Mathematics as a Gift to High School Students" as the title of the book. Initially, this suggestion surprised us. Inasmuch as our lectures to high school students were the same as those given to college students, we did not consciously differentiate them. The high school students had, however, picked up on a point that both we and our college students had missed—that education is a gift. It is a legacy that should be handed down and talked about through generations. I believe there is a similarity between the concept of learning and the concept of land as something to be inherited from those who came before us: something that can be handed down from generation to generation as a precious and irreplaceable object; I think we have lost sight of a very important philosophy that education is an invaluable gift. Something clicked in our minds that night. We decided to publish a book with the title the student suggested. The decision, which was made on the evening of December 24th, was a suitable one for Christmas Eve.

With the assistance of Iwanami-Shoten, our publisher, we are able to present this two-volume gift of mathematics. It is our hope that our readers will find the contents of these volumes to be like the joyous sound of a stream flowing from a "spring" called mathematics. We believe that this sound will be quite different from the mathematics you learned before. We believe that you will gain more than a simple joy by learning mathematics in a fun and comprehensive way. We hope you find the freedom of spirit and flexibility of mind that lie deep within rational thinking. We also hope that you will start reading this book with an attitude of optimism and finish it with a greater understanding of both mathematics and the human mind.

And now for a synopsis of the contents of this book: in the first chapter of Volume I,[1] we show that with the use of a limited number of mathematical equations, we can clarify the mysterious world of geometry hidden in curved surfaces. This clarification can be achieved by adopting a more global point of view in analyzing geometrical figures. This global point of view also will lead us towards the field called topology. This introduction to topology comes from the lectures of Shigeyuki Morita. At the end of Chapter 1, Lecture 2 is the proof of the Poincaré–Hopf theorem. The final step of this proof came from a student at one of Morita's lectures at the Tokyo Institute of Technology. This student approached Morita after the lecture and explained his proof method. Although Morita had shown a different method of proof in his lecture at the institute, he found the student's proof interesting and chose it for the book.

The second chapter addresses the topic of dimensions. You may find it interesting to know that we added this topic to the book as a result of many questions about the dimension asked at a gathering after one of our lectures. We have summarized these questions and presented them from a somewhat general point of view.

Chapter 3 in Volume II covers the journey down the road of the history of trigonometric functions. These functions were born from studying a circle. From a functional point of view they express the infinity hidden in circles. We finally reach the concept of power series expansion of functions. This road also leads to the birth of elliptic functions. This material comes from a lecture given by Koji Shiga.

The main theme of Chapter 4, which comes from the lecture of Kenji Ueno, is a geometric theorem called the Poncelet closure theorem. Among the many theorems of synthetic geometry developed during the nineteenth century, this theorem stands out for its extreme beauty of simplicity and depth. When you explore the relationship between ellipses and tangent lines which appear in the theorem from the point of view of algebraic geometry, you will arrive at the intersection point of geometry and algebra. These two distinctly different fields of mathematics influence, deepen, and expand upon

[1]Published by the American Mathematical Society as volume 19 of the series "Mathematical World" in 2003.

one another. This relationship continues to influence modern mathematics. Let us walk together down this road to a heightened understanding of the mystical properties hidden in geometrical figures. We hope that at the end of our journey you will experience the joy of learning geometry.

You can read volumes and chapters of this book in any order; they do not have to be studied in sequence. We hope you will accept and enjoy our gift.

March 1995

Koji Shiga