

## Preface

Richard Stanley has had a profound impact on combinatorics. We hope that this book gives readers an opportunity to learn some of the mathematics that he has touched.

Stanley and his PhD advisor, Gian-Carlo Rota, were at the vanguard in transforming combinatorics from a disparate collection of tricks into an organized and mature area of modern mathematics. In particular, Stanley's talent for discovering deep examples captured the attention of the mathematical community at large. This book is a recollection of this journey, putting work of the past half-century within the context of the current mathematical scene.

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Stanley graduated with a B.S. from the California Institute of Technology in 1966. He then became a graduate student at Harvard University, where he worked with Gian-Carlo-Rota (who was a professor at MIT). In 1971, Stanley graduated with a Ph.D. in combinatorics, entitled *Ordered Structures and Partitions*; the survey by Gessel in this volume discusses this work. Stanley has supervised 59 doctoral students (56 at MIT, 3 at Harvard) and mentored countless postdocs and visitors. Following this preface, we include Stanley's abbreviated curriculum vitae and a list of his doctoral students.

A hallmark of Stanley's work has been importation of ideas born outside combinatorics to crack combinatorial problems. Examples include:

- The introduction of tools from commutative algebra (local cohomology, the Cohen-Macaulay property, canonical modules, Stanley-Reisner rings, affine semigroup rings, and invariant rings) in the enumerative theory of
  - face numbers of simplicial complexes, polytopes, spheres,
  - solutions to linear homogeneous Diophantine equations,
  - partition analysis.
- Application of the hard Lefschetz Theorem in algebraic geometry, along with representation theory of  $sl_2(\mathbf{C})$  and of finite groups, to questions of unimodality and Sperner theory of posets,
- Application of the Aleksandrov-Fenchel inequalities from convexity to log-concavity questions.
- Application of symmetric function theory to partition identities, permutation statistics, and enumeration of reduced decompositions.

A wealth of combinatorics viewed through his distinctive lens appears in his books, which are paragons of clarity, organization and elegance:

- *Combinatorics and Commutative Algebra*.

- *Enumerative Combinatorics, Volumes 1 and 2*  
(fondly referred to<sup>1</sup> as “EC1” and “EC2”).
- *Algebraic Combinatorics*.
- *Catalan Numbers*.

His books do not tell the full story, however. We hope that the surveys within this volume help to round out this picture.

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This book grew, in a sense, from two conferences held in Stanley’s honor. On June 22-26, 2004, a conference titled<sup>2</sup> “A children’s party” was held at the Massachusetts Institute of Technology in honor of his 60th birthday. A special Stanley Festschrift Volume of *The Electronic Journal of Combinatorics* (Vol. 11, Issue 2, edited by Bruce Sagan) published forty papers in his honor. A decade later, on June 23-27, 2014, a conference titled “Stanley@70” was held, also at MIT, in honor of Stanley’s 70th birthday. For example, the articles of Billera and Björner in this volume were developed in conjunction with the preparation of their historical talks given at this conference.

Rather than collecting research monographs, in preparing this volume we solicited survey papers by researchers with a variety of perspectives on Stanley’s work. He also kindly accepted our invitation to contribute a short reflection on each of his own papers, for which we are extremely grateful. Within it one will find several examples of his favorite proof technique, which he calls “proof by wishful thinking”.

We were delighted by the overwhelming positive response we received from our contributors. We thank the many invited commentators, all of whom took valuable time from their schedules to help us compile this volume. We would not have been able to produce this volume without the assistance of the American Mathematical Society, and we particularly thank Sergei Gelfand, Edward Dunne, and Christine Thivierge for their efforts in this endeavor. We also thank Günter Ziegler and Anders Björner for the excellent idea of inviting Stanley to write short reflections on his papers.

Most of all, we thank Richard for teaching us the joy of combinatorics.

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Patricia Hersh (#32)  
 Thomas Lam (#43)  
 Pavlo Pylyavskyy (#48)  
 Victor Reiner (#14)

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<sup>1</sup>A standard joke in our field quotes the EC1 or EC2 exercise number discovered later to subsume one’s own favorite recently proven observation or result. Stanley’s presentation of so many ideas in this understated manner typifies his style.

<sup>2</sup>Richard P. Stanley is known to enjoy anagrams.