Index

s-t paths, 141	entry fee vs reserve price, 233
S-veto game, 196	evaluation fee, 234
ϵ -envy-free, 185	first-price, 157, 345
(λ, μ) -smooth, 152	IPL cricket auction, 232
(.,, -)	lookahead, 248
actions, 26, 105	payment rule, 258
adaptive decision-making, xxiii, 302–322	price of anarchy, 239–240
adaptive adversary, 317–319	private values, 224
as zero-sum game, 313–317	public project, 261
Multiplicative Weights Algorithm,	randomized, 236
308-311	Revelation Principle, 240–241
play zero-sum games, 311–313	revenue equivalence, 230–232
oblivious adversary, 317–319	shared communication channel, 258, 260
sublinear regret, 310	single-bidder, two items, 250
adjacency matrix, 51	single-item, 223–256
affine latency functions, 143	social surplus, 259
Akerlof, George, 82, 83	spanning tree, 260
all-pay auctions	sponsored search, 264–270, 274
Bayes-Nash equilibrium, 231	symmetric equilibrium, 229
alternative theorem, 328	trips to the moon, 270
antisymmetric payoff matrix, 30	truthful, 227, 259
approval voting	uniform price, 237
properties, 213	utility of auctioneer, 259
Arrow's Impossibility Theorem, 209	utility of bidder, 224, 259
proof, 214–216	Vickrey, 227
Extremal Lemma, 214	Wallet Game, 255
pivotal voter, 214	war of attrition, 231
Arrow, Kenneth, 219	win/lose settings, 257–277
assignment game, 296–297	with reserve price, 232
balanced outcomes, 296, 297	Aumann, Robert, 125
best outside option, 296 excess, 296	automorphism, 87
stable outcome, 296	• ,
assignment problem, 299	backward induction, 105
atomic selfish routing, 150–154	Banach's Fixed-Point Theorem, 91
auctions, xxii	bankruptcy, 188–192
allocation rule, 258	allocation rule properties
approximately optimal, 248–250	consistent, 190
Bayes-Nash equilibrium, 225, 229	monotone, 190
Bayes-Nash incentive-compatible (BIC),	pairwise consistent, 190
240	proportional division, 188
bidder model, 223	allocation rules
bidding strategy, 224	constrained equal allocations (CEA),
correlated values, 250–252	189, 190
<i>'</i>	*

constrained equal losses (CEL), 189,	Chomp, 2, 5
190	equivalent games, 22
equal division of contested amounts,	Hex, xvii, 12–16, 96
189	impartial, 3, 21
garment rule, 189	Nim, xvii, 6
Talmud rule, 190	Nim-sum, 8
bargaining, 200–204	partisan, 10–21
solution, 200	progressively bounded game, 2, 21
utopia value, 204	Recursive Majority, 20
Battle of the Sexes, 132	Rims, 8
Bayes-Nash equilibrium, 225, 229	Shannon switching game, 17
Bayes-Nash incentive-compatible, 240	Staircase Nim, 9
Bayesian equilibrium, 124	strategy, 3
Bayesian games, 113, 116, 118	strategy, 5 strategy-stealing, 6
auctions, 223–256	Subtraction, 3
Hiring a Contractor, 258	winning strategy, 3
Lions and Antelopes, 116	Y, 14
behavioral strategies, 110–112, 120	
best-response, 33, 75	commitment, 70
best-response dynamics, 77, 78, 83	common knowledge, 105, 225
bilateral trade, 274	competitive equilibrium allocations, 300
binary prediction with perfect experts,	Condorcet paradox, 206
302–304	conductance, 47
follow a random leader, 302	congestion games, 75, 87
follow the majority, 302	Consensus, 78, 83
binding commitment, 70	convex functions, 334–337
bipartite graph, 49	convex set, 35
Birkhoff – von Neumann Theorem, 61	cooperative game theory, xxii, 194–205
Blackwell, David, 320	cooperative games, 194
Bomber and Battleship, 59, 60	S-veto game, 196
Borda count	characteristic function, 194
properties, 212	Fish of little value, 199
	Four Stockholders, 198
bounded set, 35 Bouton's Theorem, 8	Glove Market, 194, 197
Braess Paradox, 138, 139	Many Right Gloves, 199
	Miners and Gold, 195
Bridg-It, 18 Program's Fixed Point Theorem 80, 02, 08	monotonicity, 194
Brouwer's Fixed-Point Theorem, 89, 93–98	shares, 195
budget balance, 273	Splitting a dollar, 196
Bulow-Klemperer theorem, 248	transferable utilities, 194
cake cutting, 183–188	core, 195–196
envy-free, 185	efficiency, 195
fair division, 184	stability, 195
	correlated equilibrium, xx, 132–135
Moving-knife Algorithm, 184 via Sperner's Lemma, 185–187	correlated strategy pair, 133
calibration, 283	correlated values, 248
	cost minimization game, 152
Centipede, 106	courses, xxiv
chapter dependencies, xxv	current flow, 60
Cheetahs and Antelopes, 67	
Chicken, 69, 84, 127, 134	decision tree, 165
Chomp, 2, 5	•
chromatic number, 80	demand graph, 291
clickthrough rates, 265	demand set, 289
closed set, 35	Demange, Gabrielle, 300
coarse correlated equilibrium, 153	dictatorship, 209, 210, 214, 216
Coloring, 83	digital goods auctions, 270
combinatorial games, xvii, 2–23	optimal fixed price revenue, 270
Bouton's Theorem, 8	revenue extraction, 271

revenue maximization without priors, 270–273	perfect recall, 110, 111
direct revelation, 240	pure strategies, 105, 110 realization-equivalent strategies, 111
	subgame-perfect equilibrium, 105, 106,
discounted payoff, 119	108
dominant strategy, 65, 224, 227	
domination, 29–31	Subtraction, 104
Driver and Parking Inspector, 66	externality, 259, 265
dual lattice, 96	extreme point, 330
Duality Theorem, 325	facility location, 155
edge, 3	fair division, xxi, 183–193
	Farkas' Lemma, 328
edge flow, 141	feasible LP, 324, 325
effective conductance, 47	first-price auctions
effective resistance, 47, 60	Bayes-Nash equilibrium, 231
electrical networks, 46, 60	uniform values, 226
conductance, 47	Fish of little value, 199
effective conductance, 47	Fish-Selling, 113, 114
effective resistance, 47	Fishmonger's problem, 243
resistance, 46, 47	fixed point property (f.p.p.), 93
employee housing auction, 281	fixed-point theorems, 89–103
envy-free, 263	Banach, 91
envy-free pricing, 291	Brouwer, xx, 89, 90, 92–98
highest, 291–294	via Hex, 96
lattice, 291	Brouwer for simplex, 93
lowest, 291–294	compact, 92
equalizing payoffs, 29, 33	Folk Theorem, 121–124
equalizing strategies, 74	for average payoffs, 123
equilibrium flow, 142	Four Stockholders, 198
Euclidean norm, 35	
evolutionarily stable strategies (ESS), 69,	Gödel Prize, 154, 286
128–132	Gale, David, 177, 300
evolutionary game theory, 127–132	Gale-Shapley algorithm, 170–174
evolutionary stability, xx	game tree, 104
sex ratios, 131	games
excess, 296	combinatorial, 23
existence of equilibrium flows, 143	extensive-form, 104–126
extension theorems, 152–154	general-sum, 64–90
extensive-form games, xx, 104–126	potential games, 75–80
actions, 105	random-turn, $161-167$
average payoff, 119	repeated, $119-124$
backward induction, 105	Folk Theorem, 123
Bayesian games, 113, 116, 118	zero-sum, 24–62
behavioral strategies, 110–112, 120	general-sum games, xix, 64–90
Centipede, 106	Battle of the Sexes, 132
complete information, 105	Cheetahs and Antelopes, 67, 83
discounted payoff, 119	Chicken, 69, 83, 134
Fish-Selling, 113	congestion games, 75, 83
imperfect information, 109–112, 118	Consensus, 78
incomplete information, 112–118	dominant strategy, 65
information sets, 109, 110	Driver and Parking Inspector, 66, 83
Large Company v Startup, 114	equalizing strategies, 74
limsup average payoff, 119	Gnus and Lioness, 71
Line-Item Veto, 105	Graph Coloring, 79
Lions and Antelopes, 116	Hawks and Doves, 127, 129
mixed strategies, 105, 110	infinite action spaces, 80
moves by nature, 113	Location-sensitive Pollution, 75
Mutual Assured Destruction, 106	many players, 71–75
perfect information, 108, 118	mixed strategy, 67

Nash equilibrium, 64, 67	infinite action spaces, 38, 367
mixed, 73	influence, 164
pure, 73	information sets, 109, 110
Nash's Theorem, 74, 89–90	instant runoff voting (IRV)
Nightclub Pricing, 81	properties, 211
payoff matrices, 67	Iterated Prisoner's Dilemma, 120
Pollution, 72, 83	
Prisoner's Dilemma, xx, 64, 83	Jensen's inequality, 337
pure strategy, 67	Jordan curve theorem, 14
Rock-Paper-Scissors Variant, 137	
Stag Hunt, 65, 83	König's Lemma, 52, 289
symmetric game, 67	Kalai-Smorodinsky bargaining solution, 204
symmetric strategies, 67	Koutsoupias, Elias, 154
Tragedy of the Commons, 80, 83	Kuhn Poker, 126
Volunteering Dilemma, 85	Kuhn's Theorem, 111
War and Peace, 65	Large Company versus Startup, 114
Generalized Hide and Seek, 298	latency function, 139
Generalized Matching Pennies, 43	lattice, 291
generalized potential function, 79	line-cover, 49
Gibbard-Satterthwaite Theorem, 210	Line-Item Veto, 105
proof, 216–218	linear programming duality, 325
Glove Market, 194, 197	linear programming, 323–331
Gnus and Lioness, 71	complementary slackness, 326
graphs, 3	dual LP, 326
adjacency matrix, 51	Duality Theorem, 326, 328–331
bipartite, 49	Farkas' Lemma, 328
edge contraction, 261	feasible set, 324
matching, 50	primal LP, 325
Grim strategy, 121	weak duality, 326
GSP (Generalized Second-Price)	link, 3
mechanism, 267–270	Lions and Antelopes, 116
77 W 101 W	- '
Hall's condition, 50	lizards, 130
Hall's Marriage Theorem, 50	Location-sensitive Pollution, 75
graph version, 51	logarithmic scoring rule, 285
matrix version, 51	Lookahead auction, 248–250
Hannan consistency, 319	lowest envy-free price vector, 292
Hannan, James, 319	Many Right Gloves, 199
Hannibal and the Romans, 45	
Hawks and Doves, 86, 127–129	market for lemons, 82, 83
Hex, xvii, 12–16, 96, 97	market sharing game, 148–150, 155
Hide and Seek, 48–52, 60	matching, 50, 51, 170
Generalized, 298, 299	matching markets, xxiii
Hiring a Contractor, 258	seller valuations, 294
Hoeffding-Azuma Inequality, 332–333	unbalanced, 294
homeomorphism, 95	maximum matching, 49, 52
Hunter and Rabbit, 52–58	maximum weight matching, 289–290
Random Speed strategy, 54	ascending auction algorithm, 295
Sweep strategy, 53	mechanism, 278
	envy-free, 263
impartial combinatorial game, 3	mechanism design, xxi, 278
imperfect information, xx, 109–112	men-proposing algorithm, 171
incomplete information, xx, 34, 112–118	metric space
individual rationality, 235, 252, 259	compact, 91
ex-ante, 235	complete, 91
ex-interim, 234, 235	Miners and Gold, 195
ex-post, 234, 235	Minimax Theorem, 27, 35–38
individually-rational payoff profile, 122	minimum cover, 49, 289
induced ranking rule, 208	minimum line-cover, 52

minimum spanning tree (MST), 260	Pick a Hand, 24
mixed strategy, 105, 110	Pigou network, 144
monopoly reserve price, 242	Pigou price of anarchy, 145
Morgenstern, Oskar, 39	Pigou-type examples, 140
Moving-knife Algorithm, 184	Plus One, 29
Multiplicative Weights Algorithm, 308–311	policy regret, 318
gains, 311	Pollution, 72, 74
play zero-sum games, 311–313	positional voting method
sublinear regret, 310	properties, 212
Mutual Assured Destruction (MAD) game,	potential function, 76, 77
106	potential games, 75–80, 83
Myerson's optimal auction, 242–248	best-response dynamics, 77, 83
Myerson, Roger, 252	Coloring, 83
	congestion games, 75, 83
Nash equilibrium	Consensus, 78, 83
mixed, 68, 69	Graph Coloring, 79
Nash bargaining, xxii, 200–203	potential function, 76, 77
Nash bargaining axioms, 200	pure strategy equilibrium, 77
affine covariance, 200	preference profile, 208
Independence of Irrelevant Alternatives	
(IIA), 201	preferred items, 289
Pareto optimality, 201	prerequisites, xxiv
- **	price of anarchy, xxi, 138–160
symmetry, 201	affine latency functions, 143
Nash bargaining solution, 201	atomic selfish routing, 150–154
Nash equilibrium, xix, 33, 64, 67, 71,	Braess Paradox, 138
73–75, 77, 83, 89, 127, 129	edge flow, 141
critique, 83	equilibrium flow, 142
mixed, 70, 73, 127, 129, 131	existence of equilibrium flows, 143
pure, 29, 68, 73	extension theorems, 152–154
symmetric, 90, 344	first-price auction, 239–240
Nash's Theorem, 74, 83, 89–90	market sharing game, 148–150
Nash, John, 83, 103, 204	network formation, 146–148
network formation games, 146–148	Pigou price of anarchy, 145
Nightclub Pricing, 81	selfish routing, 138–146
Nim, xvii, 6	
Staircase Nim, 10	smoothness, 152
Nim-sum, 7, 8	socially optimal traffic flow, 139
Nisan, Noam, 287	total latency, 141
	traffic flow, 141
node, 3	traffic-anarchy tradeoff, 146
non-transferable utility (NTU) games, 194	primal LP, 325
norm, 35	Prisoner's Dilemma, xx, 64, 119, 127
normal form, 105	procurement auction
normal play, 3, 4	second-price, 258
nucleolus, 203	progressively bounded game, 2, 4
Number Picking Game, 42	proper coloring, 79
	protein problem, 324
objective function, 324	public project, 261
optimal auctions, 242–248	pure strategy, 77, 105, 110
optimal strategies, 33	F === 0.000,, ===, ===
	quadratic scoring rule, 285
Papadimitriou, Christos, 154	quasilinear utility, 224, 279
parallel-sum game, 45	•
partisan subtraction game, 10	random-turn games, 161–167
payoff matrix, 26, 67	board, 161
payoff polytope, 121	Hex, 161
payoffs, xviii	optimal strategies, 162
Penalty Kicks, xix, xxvi	pure strategy, 162
perfect information, 118	Recursive Majority, 162, 165
perfect recall, 110, 111	selection, 161
r · · · · · · · · · · · · · · · · · · ·	

Team Captains, 161	series-sum game, 45
value, 163	sex ratios, 131
win-or-lose selection games, 164	Shannon switching game, 17, 18
ranking	Shapley value, 196–199
preference strengths, 210	Shapley's axioms
ranking rule, 208	additivity, 196
strategically vulnerable, 208, 209	dummy, 196
realization-equivalent strategies, 111	efficiency, 196
Recursive Majority, 20, 162, 165	symmetry, 196
regret, 307, 308	Shapley's Theorem, 198
rent division	Shapley, Lloyd, 177, 204
envy-free, 294	Shapley-Shubik power index, 198
repeated games, 119–124	shared communication channel, 258, 260
Folk Theorem, 121–124	signaling, 116
Iterated Prisoner's Dilemma, 119–124	simplex, 98
repeated play dynamics, 77	barycenter, 99
reported social surplus, 280	barycentric subdivision, 99
reserve price, 232	face, 98
resistance, 47	inner face, 98
resistor networks, 46	outer face, 98
retraction, 95	proper labeling, 102
Revelation Principle, 240–241	subdivision, 98
revenue equivalence, xxiii, 230–232	Sperner labeling, 102
with reserve prices, 233	single-item auctions, 223–256
Revenue Equivalence Theorem, 252	all-pay, 228, 231
revenue extractor, 271	allocation probabilities, 225, 229
revenue maximization without priors,	characterization of Bayes-Nash
270–273	equilibrium, 236–237
Rims, 8	English or ascending, 223, 257
risk-aversion, 203	expected payment, 225, 229
Robinson, Julia, 320	expected utility, 225, 229
Rock-Paper-Scissors, 129	first-price, xxii, 223
Rock-Paper-Scissors Variant, 137	i.i.d. bidders, 229
Ronen's auction, 248–250	second-price, xxiii, 227
Ronen, Amir, 248, 286	single-parameter problem, 258
Roth, Alvin, 178	Smith, John Maynard, 135
Roughgarden, Tim, 155	social choice, xxii, 206–222
Route-picking, 307	social ranking, 208
Rubinstein bargaining, 204, 205	social surplus, xxiii, 239, 259, 280, 289
	socially optimal traffic flow, 139
Saari, Donald, 219	Sotomayor, Marilda, 178, 300
saddle point, 28	spanning tree auction, 260
safety strategies, 27–28	descending, 261
safety value, 27	spectrum auction, 278, 282
Schelling, Thomas, 125	Spence, A. Michael, 83
scoring rules, xxiii, 283–285	Sperner labeling, 102
characterization, 283–285	Sperner's Lemma, 92–93, 98–102
logarithmic, 284	Splitting a dollar, 196
proper, 284	sponsored search auctions, xxiii, 264–270,
quadratic, 284	274
strictly proper, 284	stable matching, xxi, 170–176
second moment method, 332	attainable man, 172
second-price auctions, xxiii, 157	attainable woman, 172
truthful, 227	men-proposing algorithm, 171
selection games, 161	preferences determined by matrix, 174
selfish routing, 138–146	properties, 172–174
Separating Hyperplane Theorem, 35–36	truthfulness, 175
series-parallel network, 47	stable roommates, 179

Stag Hunt, 65	individually rational, 259
Staircase Nim, 9	sponsored search, 265–267
Stiglitz, Joseph, 83	envy-free, 267
strategies, 3	truthful, 259
behavioral, 110, 120	win/lose settings, 259–260
evolutionarily stable, 129	vertex, 330
mixed, 26, 67, 105, 110	Vickrey auctions
optimal, 33	with reserve price, 232
pure, 27, 67, 105, 110	Vickrey, William, 252
realization-equivalent, 111	virtual value, 246
Tit-for-Tat, 120	von Neumann Minimax Theorem, 27, 35–38
strategy stealing, 6, 12, 20	von Neumann, John, 39
subdivision	voting, xxii
Sperner labeling, 102	voting and ranking, 206–222
subgame, 105	voting and ranking rules
subgame-perfect equilibrium, 105, 106, 108	approval voting, 213
subgradient, 285, 335, 337	Black's method, 219
sublinear regret, 310	Borda count, 212
sublinear regret algorithm, 159, 308–311	Condorcet completions, 219
Submarine Salvo, 31	Copeland rule, 219
Subtraction, 3, 104	cumulative voting, 220
supporting line, 335	dictatorship, 209
surplus, 297	Dodgson rule, 219
symmetric game, 67, 75, 87, 90, 127, 128	instant runoff voting, 211
symmetric strategies, 67	plurality voting, 206
T 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	plurality with elimination, 211
Tardos, Èva, 155	positional voting rules, 212
Team Captains, 161	runoff elections, 207
terminal position, 3	voting preferences, 208
threshold bid, 259	complete, 208
Tit-for-Tat, 120	transitive, 208
top trading cycle algorithm, 176	voting rule, 208
total latency, 141 trading agents, 176	strategy-proof, 210
traffic flow, 141	voting system properties
traffic-anarchy tradeoff, 146	anonymity, 210
Tragedy of the Commons, 74, 80, 83	cancellation of ranking cycles, 211
transferable utility (TU) games, 194–195	Condorcet loser criterion, 210
Troll and Traveler, xix, 46–48	Condorcet winner criterion, 210
truthful auctions, 257–283	consistency, 219, 221
truthful mechanisms, 278–283	IIA with preference strengths, 210
truthful, truthfulness, 259, 279	independence of irrelevant alternatives
oracinal, oracinamoss, 200, 210	(IIA), 208
unstable, 176	invariance to candidates dropping out,
unstable matching, 170	219
unstable pair, xxi	minority representation, 220, 221
Uta stansburiana, 130	monotonicity, 210
utility, xxv, 157, 345	participation, 219
auctioneer, 259	reversal symmetry, 219
bidder, 259	unanimity, 208
utility functions, 71	W-1
1 4: 6 4: 070	Walrasian equilibrium, 300
valuation function, 279	War and Peace, 65
value, 224	war of attrition auctions
value of zero-sum game, 24, 27, 37	Bayes-Nash equilibrium, 231
VCG (Vickrey-Clarke-Groves) mechanism,	weighted majority algorithm, 305–306
xxiii, 259, 278–283	win-or-lose selection games, 164
envy-free or not, 263 externalities, 259	win/lose allocation problem, 258 winning strategy, 3
CAUCITIAITUICS, 200	winning strategy, o

Y, 14, 16, 23

zero-sum games, xviii, 24-62 safety strategy, 27–28 antisymmetric game, 30 best response, 33 Bomber and Battleship, 59, 60 definitions, 26 domination, 29-31equalizing payoffs, 29, 33-34 Generalized Hide and Seek, 298 Generalized Matching Pennies, 43 Hannibal and the Romans, 45 Hide and Seek, 48-52, 60Hunter and Rabbit, 52-58 incomplete information, 34, 117 infinite action spaces, 38 Minimax Theorem, 35-38 mixed strategy, 26 Nash equilibrium, 33-34Number Picking Game, 42 optimal strategies, 28, 33-34 parallel-sum, 45 payoff matrix, 26 Penalty Kicks, xix, xxvi Pick a Hand, 24 Plus One, 29 pure strategy, 27 Rock-Paper-Scissors, 129 saddle point, 28 safety strategy, 28 safety value, 27 Sequential Randomized Game, 34 series-sum, 45Submarine Salvo, 31 symmetry, 31-33 Team Captains, 161 Troll and Traveler, xix, 46-48, 59 value, xix, 24, 27, 37