CHAPTER 8

The Alter Ego

In the summer of 1890, one hundred and three high-school seniors sat for final exams at the Liceo Ginnasio Tito Livio, one of the oldest and most prestigious public schools in the city of Padua. The subjects included oral and written exercises in Italian literature and letters, as well as translations of passages from Latin and Greek to Italian, followed by more proficiency tests in history, geography, natural history, philosophy, physics, and chemistry. Topics in geometry and algebra rounded out the suite of subjects. The tests were both exhaustive and grueling; seventy-three students are recorded as having failed at least one of them *per aver interrotto le prove,* meaning they didn't make it to the end of the exam. Given the option of choosing a written exam in Greek or mathematics, only three students chose mathematics. Among them was the thirteenth candidate on the alphabetical roster, at seventeen one of the youngest in the group, and he passed all his tests—a total of eleven oral and written exams—earning a near perfect score of 107 out of a possible 110, a record few, if any, students came close to matching in that year or any other. Two generations removed from the confines of the ghetto, where nearly all Italian Jews had been sequestered for centuries, this formidable young intellect, Tullio Levi-Civita, was destined to leave his mark not only on numerous branches of mathematics but also to influence the future course of science in a manner no one could have foreseen at the end of the nineteenth century. A mathematician of enormous versatility, he almost certainly remains best known for his work on the absolute calculus and the prominent role it played in Einstein's formulation of the general theory of relativity.

Tullio Alessandro Levi-Civita was born on March 29, 1873, in Padua, and grew up in the shadow of its university, where he would teach for twenty years. Guidebooks of that era describe a picturesque city of about 66,000 inhabitants, including an ancient Jewish quarter, and many bridges, some dating from Roman times, over various branches of the Bacchiglione River flowing through the town. What the city may have lacked in modern industrial establishments at that time—contemporary records allude to a single clothing factory (a vestige of a once-thriving wool industry) with seventy workers and a lone foundry with a hundred employees—it made up for in the intensive cultivation of agricultural products and its geographical position as a hub for the growing national railway and highway system that

connected southern and central Italy with the northeastern part of the country.

No thread running through the fabric of life in the city, the historian Angelo Venture has noted, was more important than the university, which served as "the cultural metropolis"[1] of the Veneto, the region bordering the Dolomite mountains and the Adriatic Sea. Padua's "impetus to innovate [and] the push for industrial and financial initiatives" arose from the fusion of intellectual talent within the university's walls—"above all the technical-scientific knowledge of its mathematical faculty and the school of applications for engineers"—with the city's Jewish middle-class professionals, for whom emancipation in 1860 had unleashed a flood of new civic, commercial, and real estate opportunities.

The rolls of the Università Israelitica di Rovigo record the birth of Tullio's father, Giacomo Levi, the son of Abramino Levi and Rachele Civita, in 1846, and the further notice that in December 1868, Giacomo added his mother's surname to his own. Family members recall that Tullio's father, a lawyer, jurist, and politician, wanted to distinguish himself from another Giacomo Levi, no relation, who also practiced law in the same city; and perhaps he also wished to avoid confusion with his brother-in-law, yet another Giacomo Levi, who was director of the insurance company *Assicurazioni Generali,* in Venice.

Padua and its surrounding territories were part of the Austrian empire at the time of Giacomo's birth. Family tradition holds that he had, by the age of thirteen, become so vocal in his opposition to Austrian rule that his parents withdrew their young firebrand from the local Regio Ginnasio liceale S. Stefano[2] (the name of the school was changed to Tito Livio in 1872), where he ranked first in his junior high school class. He was sent to study 250 miles to the west in Piedmont, the birthplace of the movement for Italian independence, where, his family may have felt, an outspoken young Italian Jew could agitate in relative security. There he earned his laurea in jurisprudence at the University of Pavia and served as a volunteer with the charismatic general Giuseppe Garibaldi in two of the military campaigns that led to the unification of the Italian peninsula under Italian rule. He subsequently became mayor of Padua and a senator of the Kingdom of Italy.[3]

His son, Tullio, inherited his father's marked liberal bent, and although in his youth he played no active role in politics, he had the highest regard for his father and the institutions and liberties he stood for. In later years, two portraits would hang in his study at Via Sardegna 50 in Rome: one of his father and the other of Garibaldi, bearing the following dedication. "To Giacomo Levi, G. Garibaldi [Giacomo had not yet added the maternal surname]." It has been said by his mathematics colleague Ugo Amaldi that Levi-Civita inherited his strength of character from Giacomo and his capacity for empathy from Bice Lattes, his mother, whom he adored.

The house where Levi-Civita was born still stands on Via Daniele Manin 7, a narrow thoroughfare that runs between the bustling market square

and the Piazza del Duomo, bordering the former Jewish ghetto. The city's Jews had been confined to their own quarter, a warren of dark, narrow streets and unhealthy homes, for nearly two centuries, until the doors that locked them in every night were torn down in 1797 following the arrival of Napoleon's troops in the city. During Levi-Civita's childhood, the city had a small Jewish population of around 950, which declined steadily in the following decades. The Italian journalists Annie Sacerdoti and Luca Fiorentino report that 600 Paduan Jews were still living there in 1938 when the Fascist government introduced the infamous racial laws. Of these, they report in a melancholy coda, forty-seven were subsequently deported, and "only three returned at the end of the war."[4] In matters of religion, Levi-Civita later described himself as an agnostic; in a detailed curriculum vitae that he filled out for university officials in the 1930s, he made a column, labeled it "race and religion," and underneath wrote, "Jewish Non-practicing [*Ebraica Aconfessionale*]."[5]

This strain of secular thought seems to have run through his immediate family. Until the age of ten, Levi-Civita was tutored privately by Luigi Padrin, an erudite priest with an encyclopedic knowledge of medieval Padua's literary history. Like his father before him, at age ten he was enrolled by his parents in the highly regarded Tito Livio, one of the oldest and most important schools in the city. Having apparently skipped a couple of grades, he entered the second year of junior high in 1883 and plunged into the demanding academic curriculum set down by the ministry of education. In 1886–1887, during his final year as a middle-school student, Levi-Civita's weekly schedule consisted of seventeen hours of Greek, Latin, and Italian; three hours of history and geography; two hours devoted to elements of natural history, and one hour of practical arithmetic. Also housed in the registrar's files is a record of his conduct and grades, with the note "*Licenziato*," meaning that he obtained the *licenza* or diploma to enroll in the high school in the fall, with the added distinction of graduating first in his class.[6]

Levi-Civita embraced even more subjects in high school, ranging from mathematics and philosophy to nine hours a week of chemistry and physics, which he took during his senior year. A slight figure with an ebullient personality, the budding mathematician was popular with his boisterous classmates, both on the sports field and in the classroom, where his ability to effortlessly understand and assimilate new material fascinated the other pupils. Sixty years later, a schoolmate by the name of Felix Carli, who had in the meantime become a seismological engineer in Argentina, remembered the glee with which a group of Levi-Civita's school friends delighted in chanting verses composed by "my dearest Tullio,"[7] of which he could still recite the following fragment: "And when we are done with the chemistry exams... because then we really are through!" (Unfortunately his memory could no longer supply what were evidently the crucial middle lines of the verse.) In 1888, the number theorist Paolo Gazzaniga joined the high school's faculty. By then, Levi-Civita's maternal uncle, a retired engineer, had introduced his

nephew to the classic geometry texts of the English mathematicians Isaac Todhunter and George Salmon.

Eager to put his new-found knowledge to work, Levi-Civita set out to prove Euclid's puzzling fifth postulate, the parallel postulate, using only the first four postulates. In later years, Gazzaniga never grew tired of recalling how his extraordinarily precocious student confidently proceeded in a flawless and elegant way—only to arrive at a proof containing a circular argument. "At fifteen, in the disappointment of that failure, Levi-Civita could certainly not have imagined that one day his name would be attached forever to an extension, both inspired and potent, of that same concept of parallelism,"[8] Ugo Amaldi, with whom Levi-Civita would one day write a series of textbooks, remarked in his commemoration of Levi-Civita's life in 1946. Teacher and pupil would cross paths again at the University of Padua, where Gazzaniga, who collaborated with Ricci's occasional academic rival Giuseppe Veronese in the preparation of a geometry textbook for the secondary schools, also taught number theory, rational mechanics, and other topics.

Before deciding as a young teenager on a career in mathematics, Levi-Civita had evinced strong interest in classical languages as well as history, reflecting perhaps the influence of his one-time tutor Padrin, who also taught Greek and Latin for thirty years at the Liceo Tito Livio. Those interests faded as he grew older. Asked years later if Levi-Civita enjoyed reading different kinds of books, a family member replied, "I don't believe so. Or at least, only mathematics books," before adding, "Later, even though his father-in-law, Luigi Trevisani, had antique books—which I think he had bought himself, also in Latin—Levi-Civita bought only the volumes that he needed: he was absolutely not interested in necessarily collecting, as bibliophiles do, a complete set of works."[9] During his life, he did amass an impressive working collection of mathematics and physics books, well over 3,000 volumes, which his widow, Libera Trevisani Levi-Civita, donated to the Lincei Academy after World War II ended.

In his seven years as a secondary school student, Levi-Civita compiled an impressive attendance record, missing only one day of school. The story, as related to the author by historian Mariarosa Davi, the school's current archivist, concerns the last day of the school year, December 31. It fell on a Monday in the first semester of Levi-Civita's first year at the high school, and the school, which had been closed the previous week, reopened. Some students, including Levi-Civita, stayed home instead, choosing to complete the traditional vacation "bridge" between Christmas and New Year's Day. In doing so, he broke the rules and earned a 6 for conduct, almost half of his usual grade. "It was a slight deficiency, and very human, for such a good student,"[10] notes Davi. "And it makes him even more likable."

Unlike his future mentor Ricci, who continued to develop his absolute calculus throughout his career, Levi-Civita cast a penetrating spell on all of

mathematics. "[He moved] among many fields, from one to another, without difficulty—from analytic mechanics to electromagnetism, from celestial mechanics to the theory of heat, from hydrodynamics to elasticity—and everywhere addressing fundamental problems characteristic of the way he thought about them,"[11] Amaldi, who knew him well, later wrote. Somewhat unimposing physically (barely five feet tall and very near-sighted, fearlessly observing the world through glasses with exceptionally thick lenses), Levi-Civita was a mathematical polymath whose work stands out for its quality, quantity, and range. Pigeon-holing Levi-Civita "as this or that kind of mathematician,"[12] the English algebraic geometer William Hodge later wrote, is pointless. One of the many mathematicians from abroad who spent time in Rome in the 1930s, Hodge later recalled being "particularly struck by the vivaciousness and precision of his discourse"[13] and his passionate interest in all sorts of scientific questions. He added, "Viewing his work as a whole, however, the dominating impression one receives is of an astounding command of the technicalities of pure mathematics, aided by an acute geometrical intuition, applied mainly to problems of applied mathematics."[14] Although several of Levi-Civita's early papers in the 1890s clearly fall within the realm of pure mathematics, by the time he and Ricci put the finishing touches on their definitive joint article on the absolute calculus in the closing months of 1899, there was probably no one else in Italy, aside from his teacher, who could match Levi-Civita's deep understanding and facility with the subject.

* * *

When Levi-Civita, flush with youthful academic triumphs, enrolled at the University of Padua in the fall of 1890, Gregorio Ricci, the professor with whom he would forge a remarkable professional bond and life-long friendship, was embroiled, yet again, in a protracted struggle to become a full professor. It had begun two years earlier when Italy's minister of public instruction, Paolo Boselli, had declined to act on a unanimous motion of Ricci's colleagues to promote him, because under the Casati Law, a series of provisions governing the nation's educational system, Padua's science faculty had reached its allowable limit of full professors. By the following year, however, two members of the science faculty—physicist Augusto Righi and chemist Giacomo Luigi Ciamician—had accepted positions at Bologna, meaning one full professor chair (Righi's) and one associate professor chair (Ciamician's) were now available. That December the science faculty *again* voted unanimously to promote Ricci—this time to the position left vacant by Righi's departure. There was no internal competition this time, but, perhaps inevitably, a contender emerged from left field. When Adolfo Bartoli, an experimental physicist then teaching at the University of Catania in Sicily learned about the vote, he mounted an aggressive campaign of verbal and written protests to fellow academics and various officials, asking that they open a *concorso* for Righi's replacement rather than simply awarding

the post to Ricci. Naturally he had the ideal candidate in mind. "It would be a real disgrace," he lamented in a letter, "if the illustrious tradition of Padua's beautiful and richly furnished physics laboratory were entrusted to an inadequately trained scientific person."[15] According to the disgruntled Catanian's calculations, Ricci's promotion meant that "Padua would have eight full professors and only one associate professor in mathematics,"[16] which, he maintained, exacerbated the science faculty's already deplorable imbalance between the full professors in mathematics and those in the experimental sciences. Ricci, caught off guard by Bartoli's challenge and most likely experiencing an unwelcome sense of déjà vu (although Veronese, his victorious challenger in the previous go-round, was proving to be one of his warmest advocates in this one), vented his frustrations in a letter he wrote in January 1890 to Vito Volterra in Pisa. Recalling the fiasco of his first promotion attempt three years earlier and clearly dismayed at the prospect that history might repeat itself, he railed against Bartoli—"a relentless and willful person"—whose upstart meddling had become a constant thorn in his side, although he believed it also had alienated many of Padua's senior science faculty. That same day, he dispatched a second letter to Volterra, enclosing a third letter that he asked Volterra to pass on to his former teachers, Betti and Dini. What he actually wrote to them has not survived, but he likely expressed the same sentiments that he had shared with Volterra, particularly as Volterra reported that after reading it, Betti's advice was that Ricci focus on his own promotion and cease fretting about the activities of others.[17]

Matters continued to go downhill from there. That same month, the ministry of instruction's high council, apparently influenced by Bartoli's arguments, met to address the "strange disproportion" of full professors among the science faculty at Padua—more, they pointedly noted, than in most such university departments in Europe. While this did nothing to help Bartoli—Padua had by now considered and, as Ricci had more or less predicted, rejected him for its open full professorship—the council did adopt his view that the physics chair Ricci had hoped to occupy should instead be reserved for a suitable candidate in experimental sciences and recommended that the university throw it open to a *concorso.* If Padua was still bent on promoting Ricci, they suggested, this could perhaps be accomplished by transferring a full professor or two from the science faculty to the engineering school, thus keeping the number of full professors in the science department from reaching outlandish proportions. Needless to say, Domenico Turazza, the director of the engineering school, objected vehemently to this idea, as did the two hapless academics—one in ornamental design, the other in architecture—"volunteered" for the transfer. They refused to bow to outside pressure and insisted on staying where they were.

Ricci's continued frustration over this tangled state of affairs may be gleaned from a letter he received that February from his brother, Domenico, to whom Ricci had written about what he regarded as his ill-treatment.

Once again, his original letter has not survived, but in his reply Domenico wrote:

> You can't believe how distressed I felt to learn about your promotion [difficulties]. . . . It is repugnant to me to dwell too much on how great the injustice is and on the ill will they must feel towards you. Certainly it can only be explained through hidden forces and influences. . . . What is there to say, my dear Gregorio? We need to face the fact that the world is much worse than we would have predicted when we were young. Nevertheless we must not lose heart, but. . . fight with courage and persistence until the very end. I'm only sorry that I cannot help you with this fight. You say that you have influential people on your side, and I trust they are so, but in these cases there can never be too much support. [Here Domenico offers to enlist a high-ranking local official on his brother's behalf.] Meanwhile, don't let disgust and displeasure burden you; stay calm and. . . remember that even if you lose the battle, it would not, thank God, have the same consequences for you that others might face in the same situation [evidently a reference to the fact that Ricci did at least have tenure].[18]

His gloom notwithstanding, Ricci was right to believe he had influential allies. In March, the Padua science faculty rejected the high council's recommendations, voting yet again to promote him immediately and to hold a national competition for either an associate professor of physics or chemistry. Soon after, his old teacher Dini, now a deputy in the Italian parliament, complained directly to Minister of Instruction Boselli in an emotional letter, in which he addressed him using the familiar "tu" instead of the formal "lei" to emphasize their personal and working relationship. After criticizing the council's seeming determination to designate the professorial chair in physics for the experimental sciences ("a very strange vote"), Dini continued, "It would be truly painful if yet again Ricci is left behind. I could explain this and related matters far better to you in person. For now I will confine myself to saying that I believe that in all of this *cronyism* and *nepotism* [italics in original] played and continue to play a formidable part."[19] Still, to all appearances the matter went nowhere until midsummer when Giovanni Garbieri, now the only other associate professor in Padua's mathematics department, left the university for a similar position at Genoa. Within weeks Padua announced a *concorso* for his position—that of associate professor of complementary algebra, which Ricci promptly applied for and won. In November, after years of dragging its feet, the ministry of instruction's high council approved his promotion to full professor of complementary algebra. Ricci's long struggle was finally over.

What had broken the impasse? First, it seems clear that Dini's letter to Minister Boselli had had the desired effect. The minister, who had studiously held himself aloof, either out of preference or policy, from the machinations of his council, suddenly reversed course and urged Padua to proceed with Ricci's promotion. Secondly, the council's influential vice president, Luigi Cremona, who had repeatedly balked at the prospect of promoting Ricci to full professor of physics, was much more willing to entertain the idea when the chair in question was one that had always been held by a mathematician. "Ricci's titles," he opined during a discussion of the matter in October 1890, "lie more in algebra than in mathematical physics," adding that "Ricci's value as a full professor in mathematical physics could be called into question, and I would want the judgment of a commission; but his record in algebra leaves no doubt as to [his] great scientific value... in that field."[20]

As for the surplus of full professors both in Padua's science department and among its mathematics faculty, here it appears that, to use a contemporary phrase, the fix was in, and that Ricci probably knew it before he agreed to what was essentially a lateral transfer into Garbieri's associate professor position. In a late fall report summarizing the events that led up to the council's approval of Ricci's appointment as *professore ordinario di algebra,* Dini alluded to a special dispensation authorized "under article 73," in Italy's educational codes, which allowed the council to override such strictures as the maximum allowable number of full professors within a given academic department. What the government bureaucracy had previously taken away, or at least withheld, it now granted—and Ricci got his promotion to full professor. One last faculty meeting held late that year at Padua confirmed his new appointment and recommended that although he no longer held the title, he continue teaching his course in mathematical physics (the course would remain his until his death in 1925). It was an upper-division course, taught in a two-year sequence. In 1892, Tullio Levi-Civita enrolled in the class.

"[I] had the good fortune and honor to be his favorite disciple,"[21] is how Levi-Civita, decades later, would describe his rapport with the professor he likely encountered for the first time in the fall of 1890 when, as an entering freshman, he took Ricci's course in complementary algebra. There is no record of precisely when teacher and pupil—the patrician, reserved mathematician who hailed from devoutly Catholic gentry and the brilliant, gregarious student whose Jewish forebears had made the leap from the ghetto to prosperity and political prominence in barely a generation—first began to recognize their intellectual affinity; what is clear is that once they did, they forged a connection that would endure for the balance of Ricci's life. Already in that first algebra class, Levi-Civita was struck by the caliber and clarity of Ricci's lectures, writing thirty-five years later, "This course... which I attended when he gave it for the first time, has forever remained for me a model of impeccable reasoning and fruitful mathematical enterprise."[22]

Ricci's class was one of many that Levi-Civita had to take as a requirement for the laurea in pure mathematics. The courses were like mother's milk—he kept going back for more. He learned higher mechanics and rational mechanics from Ernesto Padova and infinitesimal analysis and higher analysis from Francesco D'Arcais. Theoretical geodesy he mastered from lecturer Francesco Miari-Fulcis; descriptive and projective geometry he picked up from Enrico Nestore Legnazzi. The director of the Astronomical Observatory of Padua grounded him in astronomy, using the observatory as his classroom. He found time for electives too: from classes in mathematical exercises to the theory of numbers, a subject begun in high school under the direction of Paolo Gazzaniga, who also taught at the university.[23]

In 1892, at the start of his third year, Levi-Civita enrolled in Padua's teacher-training program in mathematics, the prerequisite to an academic career at either the high school or university level. Giuseppe Veronese, who was then in charge of the mathematics section's geometry course, lectured that year on infinity and infinitesimals. He had recently produced a king-sized book on multi-dimensional geometry[24] whose methodology, using infinitely large and small numbers, had embroiled the combative professor in yet another controversy—this one centering on the notion of infinity in mathematics. Today regarded as a pioneering if somewhat uneven work that opened the door to the possibility of a non-archimedean geometry, Veronese's book, required reading though it may have been at Padua, was not universally appreciated. In particular, it drew the scorn of influential German mathematicians, who regarded it as unwieldy and synthetic. In the words of a modern German historian of mathematics, Detlef Laugwitz, Veronese's presentation "lacked clarity, and some of his formulas were obviously nonsense when one did not read the accompanying text with care."[25]

Despite these attributes, or perhaps because of them, the book inspired Levi-Civita's very first research paper,[26] "On infinities and infinitesimals as analytical elements," which he submitted for publication in May 1893, one year before he graduated. In his biographical memoir of Levi-Civita prepared in 1946, Amaldi remarks that this paper, which established the first rigorous construction of non-archimedean ordered fields, "seems even today the mature work of a proven researcher rather than the first work of an eighteen-year old."[27] (Levi-Civita in fact turned twenty that year.) Five years and seventeen papers later on a wide range of topics in analysis, mathematical physics, and mechanics, Levi-Civita would return one last time to Veronese's geometry with a work on transfinite numbers, in an effort to put to rest "misunderstandings"[28] by prominent critics of his former teacher's geometrical hypotheses.

Nevertheless, by his senior year at Padua (1893–1894), Levi-Civita had succumbed to the lure of Ricci's absolute differential calculus. While he followed Veronese's lectures on irrational numbers, took a full complement of courses, and busied himself turning out papers on assigned topics in analysis, geometry, and mechanics,[29] he also began work on his dissertation,

choosing a topic that, as he notes in the first footnote of his thesis, Ricci had suggested to him. "On differential invariants"[30] marked Levi-Civita's first contribution to the absolute differential calculus. Inspired in part by Ricci's covariant differentiation and in part by Sophus Lie's theory of groups of transformations, he extended the theory of differential invariants to more general cases than those considered by his professor. Working on the absolute calculus brought Tullio into frequent contact with its inventor, who, as their subsequent relationship makes clear, had rarely come across a student whose aptitude for study and mathematical research, taste for complicated calculations, and breadth of interests, set him apart from his peers. It must also have given Ricci considerable satisfaction that his mathematical formulations, whose relevance and utility had been questioned and even disparaged at times, had captured the interest of this exceptionally gifted student. For his part, Levi-Civita, as he would attest many years later, found in Ricci a truly admirable mentor, whose "unflinching moral rectitude, natural reserve, and the serenity with which he accepted the judgments of others equaled the power of his intellect."[31]

In July, after completing a dozen special exams with a perfect score of 30 in nearly every subject (*con pieni voti assoluti*),[32] Levi-Civita successfully defended his degree in mathematics. Soon after, the much-praised and hard-working *Dottore in Matematica con lode* ("with honors") boarded the train for Bologna, where a new generation of mathematicians trained at the Scuola Normale in Pisa had ushered in a new era of mathematical teaching and research at the university. He remained there for about six months, long enough to become more familiar with Salvatore Pincherle's research on functional operators and to strike up a life-long friendship with twenty-three year old Federigo Enriques, an instructor in projective and descriptive geometry. Writing to his colleague Guido Castelnuovo in Rome, with whom he had begun a collaboration on the geometry of algebraic surfaces, Enriques described his new acquaintance as "a talented young man and very studious, also nice personally; and for me a pleasure to be able to have a mathematical conversation with... although rarely on geometrical arguments."[33]

Returning to Padua in spring 1895, Levi-Civita was quickly pressed into service as an assistant not only to Ricci in algebra but also to D'Arcais in infinitesimal calculus and Veronese in geometry. It seemed he would not remain there long since the following year he applied for the position of *libera docenza*—essentially an instructor—in infinitesimal calculus at the University of Pavia.[34] The Pavia commission that provisionally approved his appointment expressed some skepticism about his early paper on absolute invariants, describing the first part (a presentation of Ricci's methods) "as containing complicated calculations perhaps irreproachable, but certainly not elegant, and the second part (the problem of invariant integrals) as "research that is essentially only a different form of research already noted." They were far more impressed with his more recent publications on functional analysis from the point of view of group theory, which they considered

a sign of "the author's uncommon insight and analytical ability."[35] Pavia's science faculty seconded this judgment and sent it on to the ministry of instruction's high council for approval. Only a week later, however, Ernesto Padova, professor of rational mechanics at Padua and Ricci's closest colleague in the department, died after a lingering illness. Without a moment to waste, and with no mention of his impending appointment at Pavia, Levi-Civita requested Minister of Public Instruction Luca Gianturco's assistance in obtaining the rank of *libera docenza* in rational mechanics at Padua.

Although Padua's science faculty may have dithered and delayed over Ricci's promotions, it moved with lightning speed in Levi-Civita's case. On May 16, the commission appointed by Padua's science faculty to referee Levi-Civita's publications—Ricci, D'Arcais, and Volterra—prepared a detailed report on the candidate's works. Unlike the examiners in Pavia who had dismissed Levi-Civita's approach to Ricci's calculus out of hand, this trio of reviewers merely sounded a cautious note. "Following Lie, Levi-Civita chose to study the differential invariants shared by multiple tensors, without a requirement that these include the metric tensor,"[36] they wrote. "One can perhaps argue," they added, "about the appropriateness of the choice of such an argument, which reveals moreover a tendency that appears in other works of the author, to prefer general theories in comparison to applications to concrete problems." Having surveyed the entirety of his works, the commission had only praise for the breadth of his knowledge, the exceptional ingenuity of his calculations, and the stratagems that he employed in pursuing a wide range of problems. Padua's rector then forwarded their endorsement directly to Minister Gianturco. If he was attempting to fast-track the appointment by making an end-run around the high council, whose approval was usually solicited first, he failed. The minister promptly asked his council for its assessment of Levi-Civita's qualifications. The council responded by inquiring whether the brash young petitioner intended to secure positions at two different universities simultaneously. Shortly thereafter, in early October, Levi-Civita formally notified the minister that he had withdrawn his request at Pavia, leaving only Padua's application active.

With that issue clarified, the high council gave its blessing to the appointment, singling out in particular the favorable attention that Levi-Civita's recent papers in mathematical physics and mechanics had received from scientists abroad. That November, eager to see Levi-Civita's appointment take effect as quickly as possible, university officials at Padua arranged for Eugenio Valli, a deputy in Parliament and evidently their "inside man," to speak to a colleague in the minister's office. Valli followed up with a short reminder note several days later, stating pointedly, "I would greatly appreciate if the honorable minister would *sign* the official decree immediately."[37] In December 1896, Levi-Civita received official news of his appointment. A mere two years later, in 1898, he entered the *concorso* to fill Padova's vacated chair at the associate professor level.

Although the competition began with four candidates vying for the open position, it ultimately pitted Levi-Civita against Roberto Marcolongo, a dozen years his senior and already an associate professor at the University of Messina (Sicily again! one can imagine Ricci saying in exasperation). Possibly swayed by the prerogatives of seniority, the Padua judging panel split 3-2 in favor of Marcolongo, but in giving him the green light added "that in case Professor Marcolongo does not intend to hold the position, Doctor Levi-Civita could be nominated without need of further examination."[38] This may have reflected the views of the minority commissioners, one of whom, Vito Volterra, was not shy about later stating that he had a much higher opinion of Levi-Civita. (He had previously written to a colleague that "Levi-Civita is a young man of great creativity and uncommon worth, and he is one of the most beautifully promising among young Italian mathematicians."[39]) In the end, it was Volterra's judgment that prevailed. Early in 1898, Minister Gianturco informed university officials at Padua that "to provide for the chair of rational mechanics that remains vacant [at Padua], I have nominated [as] associate professor Doctor Tullio Levi-Civita... effective, January 16 of this year."[40] Within a year Marcolongo had been promoted to full professor at Messina. In 1902 Levi-Civita, then twenty-nine, became a full professor of rational mechanics at Padua. He continued to teach there until 1918, when he went to the University of Rome as professor of higher analysis.

There is no indication that Ricci ever resented or was troubled by the fact that his former student's academic rise was both swifter and smoother than his own; in fact, as their intellectual collaboration grew, the opposite was probably true. Colleagues for more than twenty years, the absolute differential calculus was the catalyst that launched Ricci and Levi-Civita's scientific discussions, paved the way for their seminal 1900 paper on Ricci's absolute calculus and cemented a deep and enduring friendship.

Ricci's self-described disciple was not shy about invoking his mentor's methods in his research. Turning to the field of analytic mechanics in 1896, Levi-Civita published "On the transformation of dynamical systems," the first of a series of memoirs concerning the study of dynamical equations. A number of French mathematicians had already worked on the general problem of the equivalence under transformations of two systems of dynamical equations without reaching any definite conclusions. In his summary of this paper later, Volterra notes that by limiting himself to the case in the absence of an applied force, Levi-Civita transforms the original problem into a geometric one. He "was thus led to the related geometrical problem of the representation of a Riemannian manifold of any dimension by way of geometric congruences, which reduces the question to the study of the variety to which the geodesics correspond, and he distinguishes these varieties to as many types as the number of their dimensions. To each type corresponds a certain number of distinct quadratic integrals of the dynamical equations that he completes with an important theorem due to [Roger] Liouville."[41]

While Levi-Civita's explicit use of Ricci's absolute calculus figured prominently in his article, he also pointed out that Ricci's methods "up to now have not become as widely used as perhaps might have been desired."[42] Unlike Volterra, who avoided the subject, Amaldi emphasized its role in his own summary of Levi-Civita's paper: "In [his] work, the absolute differential calculus that until then Ricci—perhaps also hindered by the incomprehension of most mathematicians at that time—had used almost exclusively within the traditional boundaries of metric differential geometry, was for the first time able to demonstrate its power in the treatment of a new and interesting problem, in comparison to futile results that would have resulted from less penetrating investigations."[43]

Less demonstrative by nature than his younger colleague, Ricci never offered a public testimonial to their collaborations on the order of Levi-Civita's tribute following Ricci's death in 1925. But a letter that he wrote as Levi-Civita prepared to leave Padua for the University of Rome in 1918 leaves no doubt as to how much their personal and professional relationship meant to him:

> I don't need to repeat my feelings, which you already know, and could certainly also guess, knowing the real and profound affection I have for you and the esteem I have for your research and teaching. Since it is only human that selfish feelings will overcome altruistic ones, you may easily guess that the sorrow I feel for your distance from Padua, and for the end of our personal and professional interactions, almost outweighs the pleasure that I must feel to see one of your fondest wishes [i.e., the academic chair at Rome] granted. I cannot close this without assuring you that I was greatly moved by the extremely gracious sentiments expressed toward me that I found in your letter and for which I am very grateful.... Please remember me to your kind wife [Levi-Civita had married in 1914] and accept this expression of my unshakeable affectionate friendship for you.[44]