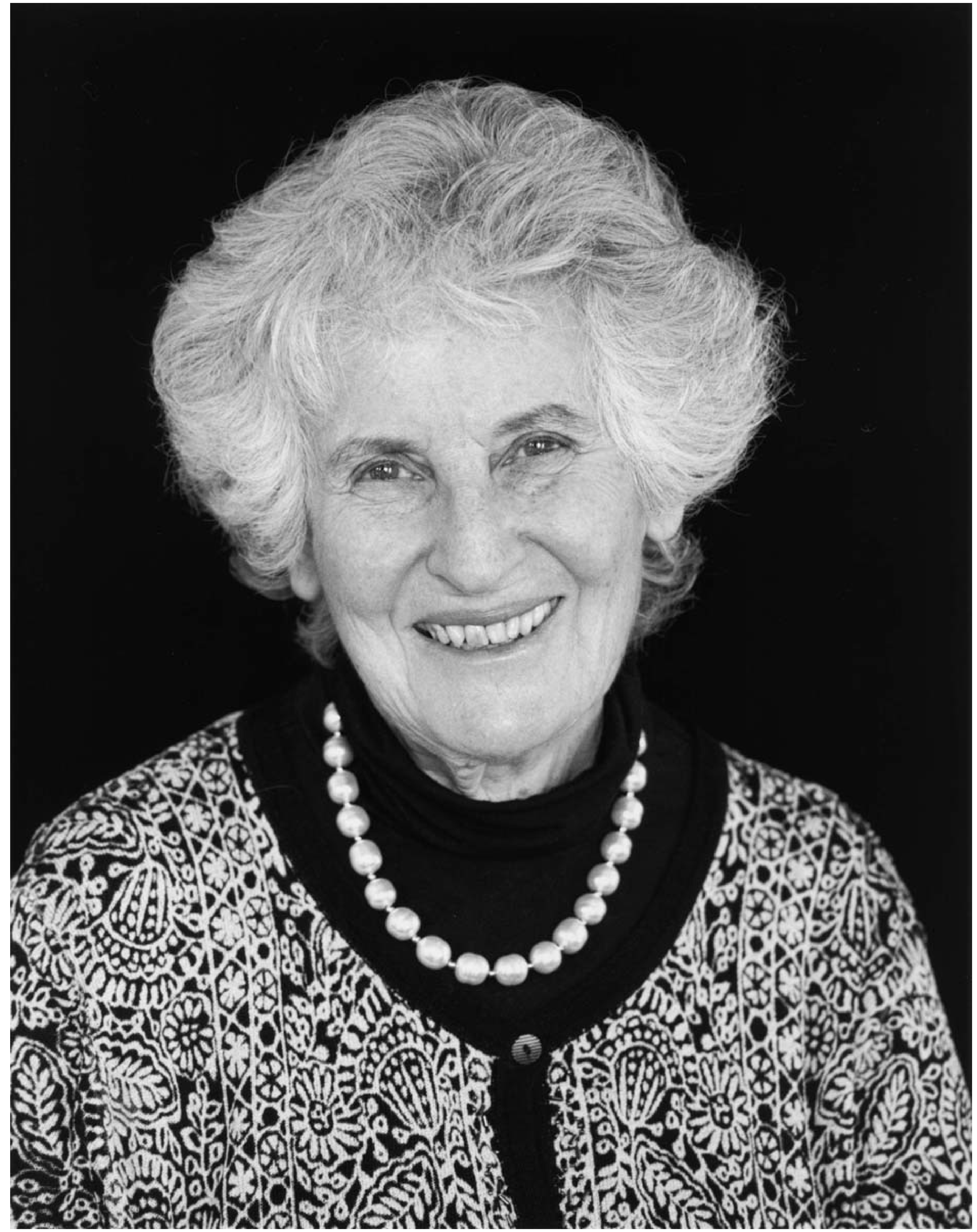
JOAN S. BIRMAN

Topology, knot theory

Professor Emeritus of Mathematics, Barnard College, Columbia University

Why did I choose mathematics? I'm not sure that "choose" is the right word; rather, mathematics chose me. As a very young child I always wanted to understand how things work: figuring out how to build a sturdy windmill that would turn without falling apart, out of rods and spools, was one example. Predicting the swirling patterns made by many rolling marbles was another. I was fascinated by such questions, enjoyed a certain kind of solitary play, and often didn't want to leave it for meals when I was called, much as I don't find it easy to stop working on a math problem today. As soon as I realized that mathematics is filled with thought-provoking questions and gives you tools for their solutions, I was drawn to it. For example, an elementary school teacher asked whether the product of two odd numbers is odd or even. What about an even number and an odd number? Why? Such questions were a challenge, and I responded to the challenge. Equally important was the fact that I did well, and to be good at something reinforces one's natural interest. So in many ways mathematics chose me, although I took many detours before settling on a career in mathematics, because life's big choices are never simple. My particular specialty within math also chose me. When I was faced with deciding on a PhD thesis topic, I did lots of hunting, but the moment I learned about an unanswered question that involved braids, I was hooked! Braids and knots are ubiquitous in nature. There are pictures in my files of braids in Saturn's rings, of long knotted loops of DNA, and even a very clear knot in a picture of the Ebola virus. More important from my viewpoint, braids and knots are also ubiquitous in mathematics.

The study of knots is part of an area of mathematics called topology. Yet here is an example, from my own work, of a way in which knots appeared unexpectedly in a part of mathematics that is far from topology, namely differential equations. In the 1960s the meteorologist E. N. Lorenz became interested in weather prediction. His belief was that weather was governed by a very large system of differential equations, and if so, it should be accurately predictable forever if one knew it at any one moment. Alas, that was far from the case, for while meteorologists know how a hurricane starts, they cannot, even with the most powerful computers, predict its long-time future path or severity with any real accuracy. Aiming for better understanding, Lorenz looked for the simplest example possible of this unpredictability and was led to a system of differential equations in three variables that illustrated the phenomenon, even though they no longer related to weather. The solutions to his equations turned out to be the paradigm for what we know today as "chaos." In my own work with R. F. Williams in the mid 1980s we learned that the closed orbits in the solution to Lorenz's equations are a vast collection of infinitely many different knots; also, any two of these knots cannot be separated without cutting one of them. This requires lots of structure, because all those knots have to fit into a smooth flow in 3-space. Now, knot theory and differential equations are very far-apart areas of mathematics, and nobody was thinking about knotting in this situation. We now understand that, loosely speaking, in any system of differential equations which governs a chaotic flow in a region of 3-space, the number and variety of knots that occur is a measure of just how chaotic the system is. The implications of Lorenz knots, is, as I write, a subject that is still being studied.



HENRI CARTAN

Algebraic topology, complex analysis
Professor Emeritus of Mathematics, University of Paris-Sud Orsay

I learned to read at home. My family lived in Paris and at six, I was sent to the Lycée Buffon. After the first test paper, I told my parents I was very proud because I came in twenty-fourth. To me this high number was a proof of excellence. It was explained that it was better to be first, which I managed to be from then on.

We spent the summer in a little village in Isère, Dolomieu, near the family smithy where my father, Élie Cartan, had spent his youth. His extraordinary gift for mathematics had been detected by his schoolmaster, and he brilliantly worked his way to the Académie des Sciences. He was a very modest man. He was probably conscious of his personal worth but never flaunted it. He did not talk me into doing mathematics, but I was always welcome to converse with him and ask him questions. I remember one day when we were walking in the woods, he told me that Euclid's postulate was not a necessity. I found it very hard to swallow! Much later we worked on a few problems together, but usually we worked on our own.

I knew from the beginning I would specialize in mathematics. To me it was the fundamental science par excellence. When I got my baccalauréat, it was quite clear to me that I would take the competitive entrance exam for the École Normale Supérieure in mathematics. Yet many things bothered me about the math I was taught. I remember that the beginnings of geometry were disconcerting to me. Unconsciously, I felt the axioms were not stated in a satisfactory way. So when I started teaching, I made sure that everything was logically coherent. I was soon reputed to be a finicky perfectionist!

I turned to my friend André Weil for advice. We were both teaching in the same university in Strasbourg. I must say that I certainly tried his patience with my questions. He decided we would meet in Paris with a few other mathematicians to work on a treatise of mathematical analysis. He would then be freed from my constant questioning. This is how the Bourbaki group began.

Our venture soon proved to be gigantic. We had to start from the basis of mathematics all over again. I loved the challenge and I was only too glad to work alongside so many good friends. In fact, most of the mathematics I learned was through Bourbaki during our joint research work. I greatly enjoyed discovering what was true and demonstrating it as simply and elegantly as possible. I also devoted much of my time to teaching mathematics, being very eager to make my students share my passion. I felt strongly that I belonged to a new generation of scientists who had to radically change the way certain mathematical theories were presented.

Beside mathematics, I am very keen on music. One of my brothers, Jean, was a very gifted composer who died early of tuberculosis. Music has always been—and still is—part and parcel of my daily life. So is politics. At the end of World War II, I was very grateful to some of my German colleagues for attempting to find news of my other brother, Louis, who had been sent to a concentration camp for belonging to the Resistance and was put to death there. I then realized that friendship between men is something that can exist independently of jingoistic prejudices. As a result of this experience, I was inspired to assume responsibilities, particularly in the liberation of Soviet dissidents. Since 1952, I have militated actively in favor of a federal Europe, free from the logic of national interests. Now that I am in the evening of my life, I still hope my dream will come true.

WILLIAM PAUL THURSTON

Topology
Fields Medal
Professor of Mathematics, Cornell University

"To understand" was the goal I gave for my high school yearbook, and this is still what drives me. I love to reach understanding: first, to see something (big or little) that doesn't make sense or is simply discordant, then to reflect and ponder, to search and stare in my mind's eye until sometimes, miraculously, vision is transformed and mist and muddle develop into form, order, and connection.

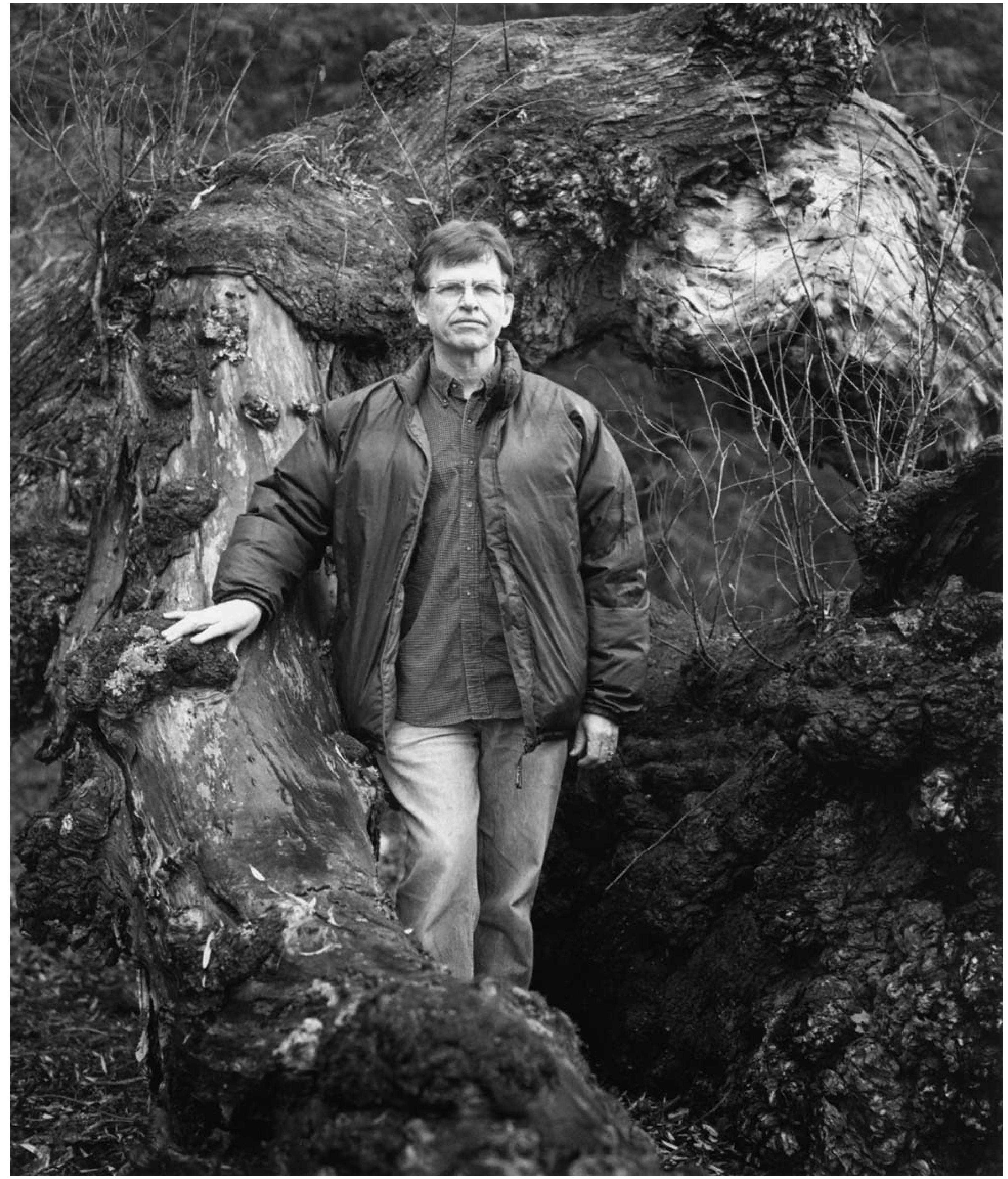
Mathematics is not about numbers, equations, computations, or algorithms: it is about understanding. I've loved mathematics all my life, although I often doubted that mathematics would turn out to be my life's focus even when others thought it obvious. I hated much of what was taught as mathematics in my early schooling, and I often received poor grades. I now view many of these early lessons as anti-math: they actively tried to discourage independent thought. One was supposed to follow an established pattern with mechanical precision, put answers inside boxes, and "show your work," that is, reject mental insights and alternative approaches. My attention is more inward than that of most people: it can be resistant to being captured and directed externally. Exercises like these mathematics lessons were excruciatingly boring and painful (whether or not I had "mastered the material"). I used to think my wandering attention and difficulty in completing assignments was a defect, but I now realize my "laziness" is a feature, not a bug. Human society wouldn't function well if everyone were like me, but society is better with everyone not being alike.

I went to a small college in the year (1964) that it started; New College in Sarasota, Florida, drawn by their discussion of educational philosophy in comparison to all the other college catalogues that I studied. This was a formative experience. There was a strong emphasis on the idea that students ultimately are responsible for their own education, there was a vision of a community of scholars including students and faculty, and there was a strong component of independent study: the initial schedule was three month-long independent study projects every year. I took these very seriously. I was very curious and ambitious to dig into the things that were mysteries. My first independent study project was titled "Language," and the second "Thought." Whether in spite of or because of their naively ambitious scope, I got a lot out of these projects, and what I learned then has significantly woven into how I work.

Mathematics has been a fantastic experience for me. I found a community of people with whom I felt comfortable. I was awed by the amazingly intricate and beautiful edifices that could be built from simple rules by pure thought. I have savored sweeping transformations continually taking place in our vision and understanding of mathematical subjects.

The biggest strand in my own work is three-dimensional geometry and topology. Imagine you are in a large cubical room, and now imagine that the front wall is identified with the back wall: in other words, when you look straight forward, your line of sight continues, jumping from the front wall to the back, and you see the back of your head. Your lines of sight continue, so what you see is images of everything in the room, repeated forward and backward as far as the eye can see. Now imagine that the left wall is identified with the right wall, and the floor is identified with the ceiling. Your lines of sight repeat in all directions, and you will see images of yourself and whatever else is in the room arrayed in a three-dimensional repeating pattern, as in the structure of a crystal. This construction yields one possible three-dimensional world (or universe), known as the 3-torus. There are many other possible topologies for a three-dimensional world. A huge variety of examples can be constructed by starting with other polyhedra, not just the cube, and identifying pairs of faces.

When I began my career, it was assumed that these three-dimensional worlds were typically hopelessly shapeless, but I gradually came to the realization that three-dimensional worlds typically come from beautiful geometrically repeating patterns, not usually in ordinary (Euclidean) space, but in one of eight kinds of three-dimensional geometries: the vast majority actually are in hyperbolic space. I formulated a conjecture, known as the "geometrization conjecture," making this precise, and I proved it in many cases. This conjecture (which includes the famous Poincaré conjecture) has now been proven by Grigori Perelman.



MARYAM MIRZAKHANI

Ergodic theory, Teichmüller theory
Professor of Mathematics, Princeton University

I grew up in Iran and had a happy childhood. There were no scientists in my family, but I learned a lot from my older brother, who was always interested in math and science. Around me, women were encouraged to be independent and pursue their interests. I remember watching programs on TV about notable, strong women like Marie Curie and Helen Keller. I admired people who were passionate about their work, and I was impressed with books like *Lust for Life*, about Vincent Van Gogh. However, as a child I dreamt of becoming a writer, and reading novels was my favorite pastime.

Later I got involved in math competitions and became more and more interested in doing mathematics. I had very good friends who were also interested in mathematics, which made my undergraduate years very exciting and inspiring. I majored in math and then went to Harvard as a graduate student. Working with Curt McMullen at Harvard, I became interested in different areas of mathematics related to dynamics and geometry of Riemann surfaces. His broad interest and deep insight had a great influence on me.

Math departments tend to be very much male-dominated and sometimes intimidating for young women. Having said that, however, I have never encountered any issue because I am a woman, and I have had supportive colleagues. Nonetheless, the situation is far from ideal. I believe women are able to do the same work as men, but the timing can be different. It might be easier for men to concentrate for longer periods of time and sacrifice more for their work. Also, what society expects from women is sometimes different from what research requires. It is very important to stay confident and motivated.

I have worked mostly on problems related to the geometry of surfaces and dabbled in related areas. Complex analysis and ergodic theory always fascinated me.

I enjoy learning different areas of mathematics and understanding the connections between them. The wonderful aspect of the problems about Riemann surfaces is the connection with so many fields in mathematics, including ergodic theory, algebraic geometry and hyperbolic geometry.

I am very slow in doing research. I don't believe in boundaries between different areas of mathematics. I like to think about challenging problems that I am excited about, and follow wherever they lead me. This allows me to interact and learn from many smart colleagues. In a way, doing mathematics feels like writing a novel where your problem evolves like a live character. However, you have to be very precise in what you say: everything must fit together like the gears in a clock.

TERENCE CHI-SHEN TAO

Harmonic analysis, partial differential equations, number theory, combinatorics
Fields Medal
Professor of Mathematics, University of California, Los Angeles

I've always liked math. I remember when I was two or three and I was wandering around with my grandmother. She was washing the windows and just to play a game with me, she'd ask me to pick a number, like 3. I'd use the detergent and she'd spray a big 3 on the window and then clean it. I thought it was great fun. I had workbooks as a kid. They were simple, with equations like $3+\square=7$. What's in the box? I thought it was really fun. Math was the only thing that really made sense to me: 3 plus 4 is exactly 7 and that's it. No one will come along later and say actually there's a new fashion and that isn't true anymore. I liked the clarity and thought of math as an abstract game to play. It was only later that I realized how it related to the real world and how it's used for all kinds of things.

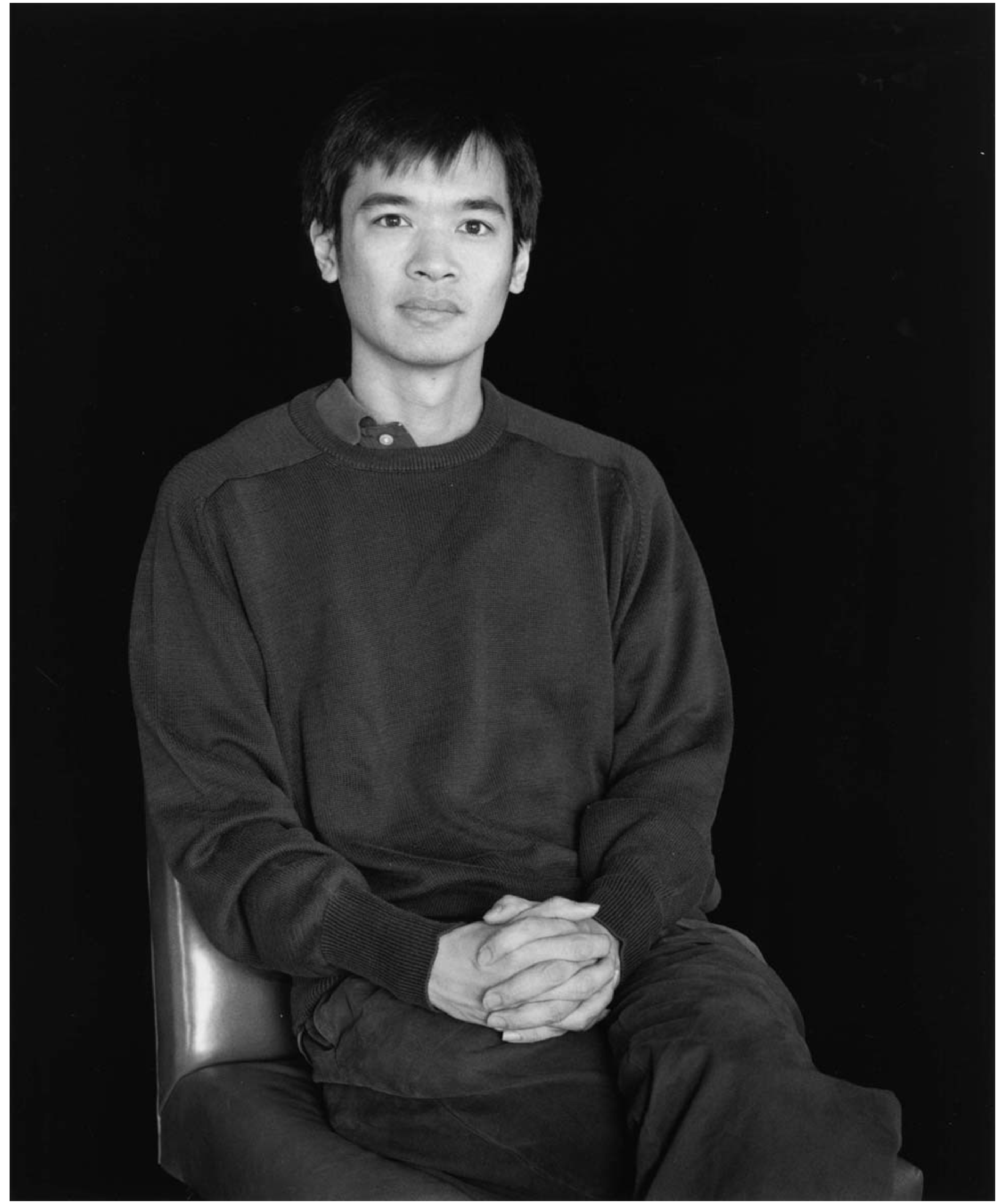
I grew up in Australia. My parents had me tested as a child, and once they realized I had some ability, they arranged special classes for me. I skipped some grades, though in a staggered way. For instance, when I was in eighth grade, I'd do some classes in English and physical education but I was taking twelfth grade mathematics and eleventh-grade physics. When I was in twelfth-grade in high school, I'd take college classes in math. My mother had to pick me up from high school and drop me at the local university. It was very complicated. In some classes I'd be with people roughly my own age and in other classes I'd be with people five years older. Most of my classmates were much taller and bigger than me. It was a shock when I taught my first class at UCLA at age twenty-one because for the first time ever I was the oldest person in the room.

I study the prime numbers. These are numbers which can't be divided by any other number except themselves and 1, like 2, 3, 5, 7, 11, and so forth. One of the things I showed with Ben Green is that you can find a certain type of pattern inside them known as an arithmetic progression. Somewhere inside these primes you can find five primes or ten primes or twenty primes or as many primes as you want, which are equally spaced. Prime numbers have been studied for three thousand years, mostly out of curiosity. The average person in the street doesn't need these prime numbers for anything. But the funny thing is that about thirty or forty years ago, it was discovered that prime numbers were very good for cryptography; in fact, they were much better than the other codes that people had invented. Nowadays, if you use an ATM machine or a credit card over the Internet, they scramble all your data by a certain code which is based on properties of prime numbers, because they are one of the most secure codes we know.

Mathematics can be like archaeology. You might find a corner of something and decide it's of interest. Then you dig somewhere else and you find another corner that looks very similar and think there may be some deep connection. You keep digging and finally discover the structure underneath. You have a thrill of discovery when something finally makes sense.

I work with a lot of very good and intelligent people and I've learned a lot from them. But it doesn't pay to take the attitude that you have to be a super genius in order to succeed. If one springs a math problem on a lot of really good mathematicians unexpectedly, they'll initially be slow in responding. You can watch them thinking. After five or ten minutes, they'll come up with some really good suggestions. They might not be very fast but they can be very deep. Everyone's got different skills. It's like athletics. There are sprinters and there are marathon runners. A sprinter would be horrible as a marathon runner and vice versa, but they're both good talents.

There are a lot of problems I'd like to solve in my lifetime, but many of them are like cliffs and I have no obvious way to get up them. I'm working on things that are more within my reach and I'm hoping to accumulate more tricks and tools and insights. Then I'll go back to the problems I really want to solve and see if anything has changed. Occasionally they budge a little. It's a bit like fishing. You can be a good fisherman and you can be in a place where there are lots of fish but you still have to wait for the bite.



MARIE-FRANCE VIGNERAS

Algebraic number theory, Langlands program
Professor of Mathematics, Institut de Mathématiques de Jussieu, Paris

I was raised in Senegal. I mention it only because I received a prize years later by proving that "one cannot hear the shape of a drum": in a mathematical sense there exist distinct drums which cannot be distinguished by their sounds. The question was raised while I was attending a conference in California in 1977 and it reminded me of nights in Africa when I listened to Senegalese people playing drums and dancing outside our home; I had tried to guess what the instruments were from their sounds.

Happy coincidences like this one are often responsible for theorems. Ideas come while lying in bed or sitting in a lecture or in a concert, when there are no worries, no teaching or cleaning, no stress. Think of yourself in a forest. You enjoy the beauty of nature and it is not cold, but light becomes dim and it is time to leave the forest. You try a tiny path but it ends quickly. You walk back and try another one; they all look the same and it is darker. You stop and stay motionless. You wait and wait, with your senses alert to see the invisible, to feel the undescribable, to listen to the silence. And it happens suddenly: one direction becomes more dense, or more luminous. To experience this intense moment is the reason why I became a mathematician. Energy, concentration and hard work are needed to write proofs. You have to pay attention. Mistakes can easily slip into them. As mathematicians, we play and dream but we don't cheat. You can't cheat in mathematics. Truth is so important. To solve a problem with a proof is exciting and rewarding because it is true forever.

I became a mathematician by luck (starting with a good teacher in Dakar, Damon, and the excellent French education of the time) and I am living a simple life. I teach for four months and do mathematics during the rest of the year. I love both. Mathematics is the deepest thing in my life and it influences me strongly. I feel I am different from, let us say, my neighbors, but not so much from historians, writers, poets, and artists.

ADEBISI AGBOOLA

Number theory, arithmetic geometry
Professor of Mathematics, University of California, Santa Barbara

Unlike many other mathematicians that I know, I was not enamored of mathematics as a young child. I found it dull, confusing, and difficult. I was interested in, and good at, most subjects in school, but I had no interest at all in mathematics—despite being constantly told by my parents and teachers how important it was to acquire a good knowledge of the subject—and for years I regularly failed almost every mathematics examination that I took. I remember on one occasion when I was very young, I decided that I hated mathematics so much that I was determined to think of a career that I could pursue that would involve the use of no mathematics whatsoever. My parents were understandably quite skeptical about this, and when I triumphantly announced "wood cutter," they pointed out to me that this would not do because I would in fact have to measure the wood.

This situation changed completely when, at about the age of twelve or so, in my school library, I came across one of the volumes of the Life Science Library (published by Time–Life International) entitled *Mathematics,* by David Bergamini. This was unlike any other mathematics book that I had seen before. It was essentially an account of the history of some of the main ideas of mathematics, from the Babylonians up until the 1960s, and it captured my imagination and made the subject come really alive to me for the very first time. After reading this book, I knew that this was what I wanted to spend as much of my time doing as I could. I became fascinated by mathematics, and I found that I enjoyed it enormously. This is what led to my deciding to become a mathematician.

Later on, after I had completed my undergraduate studies at Cambridge and it was time for me to think about beginning research towards a PhD, I became interested in number theory. For me, one of the most beautiful things about mathematics is how several different notions or ideas that at first sight have nothing at all to do with each other can in fact be shown to be closely related, sometimes in very profound and mysterious ways. This phenomenon occurs frequently in number theory, and it is one of the main attractions of the subject. It has been suggested that in some respects, pure mathematicians have more in common with creative artists than with hard scientists. I think that many number theorists especially, myself included, would agree that there is a great deal of truth to this statement.

I am sometimes asked, especially by students, how I go about deciding what topics or problems to work on. This is a hard question for me to answer precisely. Some mathematicians explicitly decide that they want to solve a particular problem, or they set out to develop a large research program in a certain area. I do not work in this way. In my case, what tends to happen is that I find that I am curious about certain things at any given time and I want to understand them better. (Sometimes these are things that are well understood by some other people, but not by me.) I go to talks and seminars, I read papers, I talk to people and I ask myself questions, I play around with examples, and in this way one thing leads to another and new ideas emerge. I should also point out, however, that most of the ideas that I have and the things that I try do not work, and I suspect that the same is probably true of many other mathematicians. Of course this just means that perseverance is a crucial part of the entire process, and that it is very important not to just give up too easily!

AFTERWORD BRANDON FRADD

This book came about because I had the wonderful opportunity to meet Mariana Cook and her husband Hans Kraus. We hit it off! Mariana sent me a book of photographs of famous scientists she had published and I immediately asked if she would do one of mathematicians. She loved the idea!

Great mathematicians have usually been considered rather different from the rest of the population, but of course they are people just like everyone else. I think that to be great at mathematics, you probably do have to have some differences: an ability to concentrate in a certain way, detach from the distractions of the environment around you, and envision possibilities that others might label "crazy." Still, they are regular people, too, who marry, form clubs, vote, and push their kids on swing sets. How different are they? How could we even tell? This beautiful book answers some of these questions. In these portraits, you will see how similar mathematicians are to us and how they are different from one another. You will have a glimpse of each personality through the expressions and the poses. The photographs along with the essays will give you an idea of the unique qualities of each individual. The essays have been carefully written either by the mathematician or Mariana Cook, based on personal conversations she had with them. They are all different and each captures something special.

There are many reasons why I asked Mariana to take these photographs. I have always loved math. I love figuring out a problem. Actually, the problem grabs me and I can't get away from it. I am obsessed by the problem until I figure out the solution. I love seeing relations among things. I can't help it. It just seems to be who I am. I always knew I would major in mathematics in college. I started first at Columbia and then finished at Princeton where there were many legendary people, and lots of "can you believe this?" stories. Burt Totaro was the youngest student admitted as an undergraduate to Princeton and John Milnor won every math prize possible. Then there was Charles Fefferman, who got tenure at Princeton when he was 24! These people loomed large for me the way baseball stars or rock stars might loom for most other people. A part of me wondered what it was about them that let them see so much farther than I did. I spent an afternoon with André Weil, who revolutionized the field of algebraic geometry and when I asked him the secret to his success, he said "at an age younger than I was supposed to be I understood things I wasn't supposed to be able to understand." Translation: he didn't know any more than I did how he did it. The mystery was on both sides.

I went on to medical school and then to investing. I have spent most of my career running a biotech hedge fund. I bet on whether new products will succeed or fail in clinical trials or get FDA approval. I use many of the conceptual ideas I learned while studying mathematics to do my analysis. Many times this approach lets me make a very strong conclusion in advance of the data. In these cases, I can take a large bet and over the 13 years I have run the fund, I have had some real financial success. Still, a piece of me always wishes I had had the ability to be a great mathematician, the kind of mathematician included in this book. I think that there are many people like me and I hope that this book will inspire others to take another look at math and see it for its possibilities and excitement. We need to get beyond the stereotype that it is "hard" and only for a few special people.

There are many wonderful people who have been involved in the development of this book. Rather than thanking them individually, I send thanks to everyone collectively, knowing that you know who you are. Thanks!

LIST OF MATHEMATICIANS