INTRODUCTION

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1 Welcome

Welcome to the third edited volume on the topic of mathematics and fiber arts (The first two volumes are *Making Mathematics with Needlework* [2] and *Crafting by Concepts* [1].). This third volume is similar in that it brings together a collection of mathematicians, each of whom will explain a specific mathematical idea or set of ideas and how the idea(s) can play out in a fiber-art setting. Each chapter concludes with a project to help realize the mathematics presented in a fully integrated and tactile way. Someone with a different perspective might say that each chapter introduces you to a fiber-arts project that uses mathematics in a novel way. The text of each chapter is written to help you comprehend how the author sees mathematics playing a role in the fiber arts presented. In many cases, the projects are presented in such a way that, based on the instructions given, the reader can create multiple variants of the design, with the reader's imagination the limit.

This book has been written for you, and it is our greatest hope that you will enjoy it to its fullest extent. Act like a mathematician as you challenge the book, talking back to what the authors say. Grow and learn as the book challenges you!

2 Mathematical Fiber Arts

As we have been involved with the development of the field of mathematical fiber arts over the past decade and a half, we have been asked the question more times than we can count, "What is mathematical fiber arts?" Without wanting to limit the scope of the field, we illuminate four of the primary types of work that are present in the field:

1. Using mathematics to solve fiber-arts problems,
2. Designing and fabricating a fiber-arts piece to display a particular mathematical concept,
3. Proving that a specific fiber art can be used to exemplify a mathematical concept,
4. Mathematically analyzing an aspect of a specific fiber art.

These types will be elaborated upon in the remainder of this section. Each chapter of the book will be identified with one type of work, although we note that it is common for an author to exhibit aspects of more than one type of work in a chapter. Additional examples of the type of work will be drawn from the books *Making Mathematics with Needlework* [2] and *Crafting by Concepts* [1]. We alert readers that many of our examples use technical terms not defined here. Be undeterred! Readers unfamiliar with the terms should still be able to comprehend the broad ideas.

Needleworking mathematicians commonly find themselves using mathematics to solve simple fiber-arts problems to adjust sizing of garments. This is an especially common issue for knitters using the concept of gauge, which has to do with the number of stitches knit per inch. In Chapter 5, Givens takes gauge calculations to a new level, carefully solving an exasperating problem of the stitches-per-row adjustment when gauge changes. This hidden delight is part of a chapter on using the Chinese Remainder Theorem to calculate feasible numbers of stitches when knitting distinct rows using stitch patterns with stitch-repeat lengths. Yackel, in Chapter 3, attacks a fiber-arts challenge using ideas introduced by mathematical artists. In doing so, she creates a set of patterns that she then counts using combinatorics. Shepherd shows the reader an algorithm for creating certain kinds of puzzle quilts in Chapter 4. A mathematical background is not necessary to follow Shepherd's directions; her use of graphs to derive a solution is lovely.

Avid needleworkers, who are also mathematicians, often want to meld these two loves, using their fiber-arts talents to make visual the abstract mathematical concepts they study. Doing so can bring to life for those around them the mathematical objects of their thoughts. Wildstrom's pentominoes in Chapter 1 can be configured into decorative wall-hangings or blankets. These invite the viewer to envision reconfigurations, which in turn raises interesting mathematical questions. In contrast, Calderhead's interwoven crochet Gosper curve, in Chapter 2, stuns the viewer with both the fiber-arts technique and the intricacy of the curve achieved. Both Gould and Nimershiem create fabric surfaces. Gould constructs finite sections of regular infinite polyhedral surfaces in Chapter 7. In Chapter 8, Nimershiem illustrates that the complement of the Borromean rings in S^3 is the union of two ideal hyperbolic octahedra. Moreover, Gould's and Nimershiem's fabric models serve an additional purpose by allowing the public to physically interact with the models, gaining further understanding.

In the case of fiber arts, the selection of technique can be especially important to create a natural correspondence between the mathematical principle one is trying to expose and the finished object. This type of work has been separated out from design and fabrication because of the intricacy involved in verifying that the fiber-arts technique aligns with the mathematical concept. In Chapter 6, belcastro introduces a technique, which she calls an algorithm, for knitting certain classes of knots, and she proves that her physical knitting techniques work. An excellent set of examples of this correspondence is displayed in Ted Ashton's chapter in *Crafting by Concepts* [1, ch. 4] about depicting the Sierpiński gasket, a fractal shown at stage four in Figure 1. He suggests five different techniques for creating a finite stage in the building of this fractal, which we will refer to as the Sierpiński triangle and thereby abuse terminology. Using cutwork, one can physically cut triangular pieces out of cloth, leaving, as the remainder, a Sierpiński triangle. The cutwork technique corresponds to the method of conceiving of the Sierpiński triangle as the limit of an equilateral triangle where at each stage the middle triangle of each solid triangle is removed.

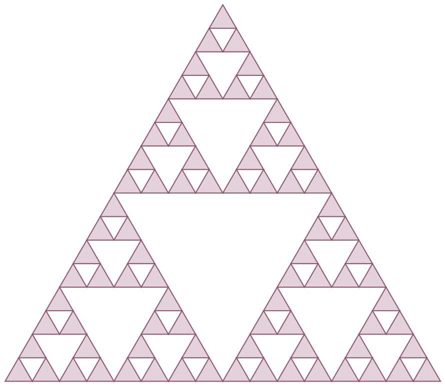

Figure 1. The Sierpiński triangle.

Gluing or sewing beads onto fabric to represent the location of points reached in the Chaos Game [3] yields a pixelated-looking version of the Sierpiński triangle. String art, in which colored threads are wrapped around nails in a specific order, can also be used to approximate the Sierpiński triangle, here taking advantage of the realization of the boundary of each stage as an Eulerian cycle. One may cross-stitch colored squares to show cellular automata, starting with a single cell on one row and applying a preselected rule on each subsequent row. By applying one of the rules that yields the Sierpiński triangle, such as rule 18, the triangle emerges as a cellular automaton. Finally, tatting the Sierpiński triangle clarifies for the fiber artist the self-similarity of the fractal, because the directions call for the tatter to tat three copies of stage n to complete stage $n+1$. The understanding that the same mathematical object can be obtained from each of these abstract starting points is nontrivial. The matching of these five fiber arts techniques with the corresponding mathematical aspects of the Sierpiński triangle is brilliant.

The fourth type of work mentioned at the beginning of this section, mathematically analyzing an aspect of a specific fiber art, did not drive any of the work in this volume, but it has motivated a great deal of work in the mathematical fiber arts. Some of the work surrounds examining the natural symmetries that arise through a given fiber art. Susan Goldstine's 2017 Bridges paper entitled *A Survey of Symmetry Samplers* [4] summarizes that work. An often-cited piece is Shepherd's chapter in *Making Mathematics with Needlework* [2, ch. 5] showing that only 12 of the 17 wallpaper patterns can be created in cross-stitch on a square grid. An investigation not motivated by enumerating symmetries is captured in belcastro's chapter in *Making Mathematics with Needlework* on the possible knit (and purl) stitches [2, ch. 4].

The four types of work described here continue to be reflective of the field of mathematical art. We look forward to more investigations and artistic pieces along these lines to emerge from the vibrant community of mathematical fiber artists across the globe.

3 How to Use and Appreciate this Book

This book has been designed to be visually stimulating. More than that, the authors have endeavored to communicate deep mathematical ideas through clear exposition and visual displays. Each chapter can be read independently of the others. Crafters may wish to first choose a chapter by fiber-art type and then by project, whereas we imagine mathematicians may select a chapter based on mathematical content. To aid this process, a mathematical abstract and a nontechnical abstract has been included for each chapter. These abstracts are located in the abstract section at the front of the book.

Once you choose a chapter to study, we hope you will read Section 1, which is an introduction to the chapter. These sections contain both relevant mathematics for crafters and necessary craft knowledge for mathematicians so that all audiences can glean the most from the chapter. Further sections of each chapter contain the mathematics and are written for mathematicians. Crafters, we urge you nonetheless to attempt these sections with the knowledge that they will contain tremendously useful diagrams. The final sections contain the instructions for the chapter project.

To have a relaxed approach to the book, you should know that mathematicians read slowly and in careful detail, studying texts for hours. You may find the reading slow going compared with typical craft books or novels. The need and desire to study is positive rather than negative. We hope you enjoy you the time you spend with this volume.

Importantly, making the chapter project can help the reader to comprehend the chapter mathematics on a far deeper level. This fact leads some people to think that mathematics can be taught through the use of fiber arts without simultaneously teaching the mathematics. On its own, crafting will not lead students to automatically understand the deep mathematics that trained mathematicians, who are also practicing fiber artists, are sometimes able to articulate with a great deal of careful investigation. Instead, interaction with the chapter projects in this book will whet crafters' appetites for the mathematics, arousing their curiosity and readying them to investigate the mathematical intricacies explained in the chapter. Conversely, the student of mathematics who takes the time to make the physical objects will add to their understanding, as the struggle to conjure abstract concepts into reality takes place.

We remind all consumers of mathematics that the most difficult part of mathematics comes when no one is looking: in conceiving of the ideas to investigate, in framing the problems correctly, in forming the arguments from a perspective that lends elegance, in checking the details, and in writing well with good diagrams. Often good expositors are robbed of the credit of their deep work because their clear explanations keep readers from realizing the heavy lifting the authors preformed for the audience. We trust that you will recognize the significant contributions of our authors to the body of literature.

We thank you for this opportunity to introduce you to some of the exciting work in the field of mathematical fiber arts. Again, welcome.

Bibliography

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