Preface

What This Book Is About

As we near the end of the second decade of the 21st century, humans are grappling with a number of tough decisions concerning our place in the natural world and the consequences of how we have used various resources and impacted our environment. For instance, human populations have grown in many areas to the point that the habitats of wild species are being lost, and those species are being driven to extinction. We have continued to burn fossil fuels to generate much of the energy for our industries and transportation, with undesirable effects from the by-products and pollution that burning generates. It is growing increasingly clear that this human activity is causing long-term changes in the Earth's atmosphere and climate. New diseases have entered human populations and spread as humans have penetrated previously unsettled areas and traveled more freely around the Earth. Invasive species brought by humans into new habitats are upsetting ecological relationships that evolved over long periods of time. Can we coexist with the other life forms that have evolved on this planet? Are there realistic alternatives to fossil fuels that would sustainably provide for human society's energy needs and have fewer harmful effects? How do we decide what alternatives make more sense? How do we deal with threats such as emergent diseases?

As a mathematically literate and thinking human being, I firmly believe that our ability to develop answers to such questions—and to understand the political, economic, and social issues involved—depends on being able to deal in an informed way with *quantitative information.* Mathematical models—equations of various sorts capturing relationships between variables involved in a complex situation—are fundamental for understanding the potential consequences of choices we make. Extracting insight from the vast amounts of data we are able to collect requires analysis methods and statistical reasoning. Understanding the basics of the mathematical tools involved is necessary to evaluate conflicting claims and recommendations for action. So, even more so than in the past, basic mathematical literacy is, I believe, an important component of informed citizenship.

In this book, we will introduce a number of basic techniques for constructing models and analyzing data. We will see ways they can be applied to study our place in the natural world and environmental issues. More specifically, we will study the following topics:

(1) basic techniques of measurement, data analysis, and presentation of data in numerical and graphical forms,

(2) functions and modeling: we will see how to use linear, exponential, and power functions to describe different situations and how to select an appropriate model for a given situation,

(3) difference equations and modeling: we will see how to set up, and solve difference equations that describe how systems evolve over time (treating time in discrete units),

(4) the use of various sorts of descriptive statistics to, understand the patterns in data,

(5) the basics of probability,

(6) the statistics of sampling from a population, and

(7) hypothesis testing: we will see how scientists use statistics to demonstrate that their data either supports a conclusion or interpretation or does not do so.

In the main text, we will not make use of any calculus or mathematics more advanced than ideas about functions, graphs, algebra, some geometry, etc. "Technical Notes" in footnotes and in optional sections in some chapters, plus a few clearly marked exercises, will deal with connections with more advanced mathematics for readers who have more background. All the mathematical prerequisites and some of the basic ideas behind material we study should be familiar from mathematics courses in the standard high school curriculum. What will probably be different, though, is the consistently applied and frequently environmental focus of everything we do.

Readers of the older text *Quantitative Reasoning and the Environment* by Greg Langkamp and Joseph Hull (Prentice Hall, 2007) will recognize the influence of that work on the development of this book: for the general idea of an elementary text on mathematical modeling and data analysis focused on environmental applications, for the choice of topics and, in particular, for the ideas behind some of the chapter projects. While the general outline of this book is close to theirs, our treatment of the mathematical topics (especially in the statistical chapters) is substantially more complete and at a somewhat higher level of mathematical sophistication. But I believe this is still appropriate for introductory courses with no prerequisites beyond high school mathematics. The examples here are also substantially more up-to-date, taking developments since 2007 into account.

However, as you surely know, understanding the predictions of even the most accurate and realistic mathematical models is *not the whole story*, not by a long shot! Some of the questions raised above have been at or near the center of the political polarization in the U.S. that has become more and more pronounced over the last 20 years.

Andrew Revkin, the writer of the former *New York Times* blog *Dot Earth* attributed the following insightful comments to a reader named Oran Switzer in a post from August 31, 2011:[1]

> There is not one climate dispute. There are two, and the solutions are not the same. First, we need to separate the two. The science debate does not work in politics. If you study the conservative approach to climate change policy long enough, the implication that they are trying to participate in a scientific conversation starts to fade away[2] and you realize the underlying logic they are using actually starts from the conclusion that regulation and government intervention are bad[3].... This allows them to make big, bold, statements about their identity and character and values rather than wallowing around in overly-precise, overly-pedantic language and data.
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> ...[W]e need to spend less time on the details of the scientific debate and much more on the underlying values—the costs to humanity, society, and the economy of extreme weather, local floods, local droughts, freshwater scarcity, infectious disease, food security, coastline loss, biodiversity loss, etc. It sounds backwards since the political challengers are denying the possibility of those dangers, one might think we need to respond to their challenge.
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> We do not. That's what science is for.

The main point here is that the insights gained from science must also be communicated in an understandable and down-to-earth way to provide the material for informed decision making.

Ask yourself, are you a communitarian or an individualist? Are you a liberal, a conservative, or perhaps a libertarian? Do you believe that climate change is a "hoax" or is it a real problem that we must address in order to ensure a decent standard of life for our descendants?

While I have my own views about these questions (and you will no doubt see what they are as you read), *I will never try to impose those views on you.* This means, for instance, that we will examine the data about carbon dioxide levels in the atmosphere using mathematical tools and examine what scientists have determined about the results of changes in the atmosphere. But I will not try to convince

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[1] A. Revkin, "The Role of Values in Driving Climate Disputes," `https://dotearth.blogs.nytimes.com/2011/08/31/the-role-of-values-in-driving-climate-disputes/`, accessed July 10, 2018.

[2] Even when scientific language is used, the main goal often seems to be to create doubts in the minds of ordinary people.

[3] For example, this was essentially President Trump's argument for pulling the U.S. out of the Paris climate accords in June 2017. He claimed that the accords would lead to a loss of independence for U.S. businesses in their decision-making and unfairly hamstring them in competition with the rest of the world.

you about any specific proposed action to counteract this. Similarly, we will study how radioactive decay works using a mathematical model, but I will not try to convince you either way about whether nuclear power generation is a reasonable alternative to burning fossil fuels. The goals of this book are to show you the light mathematics can shed as we try to understand human beings' place in, and effects on, the natural world and to help you develop your own thinking about these issues through an understanding of what science and mathematics can tell us.

Comments for Students and Instructors

I strongly believe that understanding mathematics requires one to learn to read mathematics and to *do some mathematics*. Much of this book is structured like other mathematics textbooks you may have used. This means in particular that its text is more highly organized than the text of most other similar books. You will need to keep the following signposts and cross-references in mind as you read.

- Propositions and Examples are numbered within each chapter in one sequence. Figures are also numbered within chapters, but in a separate sequence. So, for instance, if a reference is made to Figure 5.3, you should look for the third figure in Chapter 5. Proposition 5.3 happens to appear on the next page, but that is a coincidence. For typographical reasons, a figure might not appear on the same page as the text that refers to it. This is why the numbers are important—they should help to guide you to the proper figure to consult.
- Some of the equations set off on separate lines are also numbered to make it possible to refer to them within the text. These equation numbers occur at the left of the line and occur in one sequence within each chapter. A reference to (4.5), for example, means to refer to the fifth equation within Chapter 4.
- The ends of the few proofs in the book are marked with the $\square$ symbol. Ends of examples are marked with $\triangle$ symbols.
- Tables and footnotes[4] are also numbered within chapters.
- References to books, articles, and online information used in the text are collected at the end of each chapter. These are indicated by references in the text or footnotes of the form [**3**]. References are numbered in separate sequences within chapters, and the numbering corresponds to the order of the references within the text (that is, the lists of references at the end of the chapter are not alphabetized).

There is a section of Exercises at the end of each chapter except Chapter 3. These are an integral part of the book because those problems are where you will get the chance to test your understanding and practice what you have learned. Many exercises extend the discussions of the main part of the text to new situations and/or introduce new real-world examples.

Finally, every chapter concludes with an extended chapter project (Chapter 3, in fact, is entirely devoted to one of these projects). Depending on how your course is structured, your instructor may assign some or all of these as individual or group projects. Many of these involve substantial computing work either in spreadsheet

[4]This is a footnote.

software (such as Excel, Google Sheets, or LibreOffice Calc) or in the R statistical package.[5] Here the goal will be to explore a topic in depth using mathematical tools, understand patterns, apply techniques learned earlier, and then to produce a report clearly explaining what you did and the results you found.

Comments for Instructors

This book on elementary topics in mathematical modeling and data analysis is intended for an introductory-level undergraduate "liberal arts mathematics"-type course but with a specific focus on environmental applications. Those applications are the main point of the book, in fact, because I believe that a solid level of quantitative literacy is a prerequisite for anyone who wants to be an informed participant in some of the most important and consequential policy debates in progress at present.

I have used this material in seminar courses for first-year students at my home institution several times. These courses have also served to satisfy a quantitative course requirement for my institution's Environmental Studies Concentration. The learning goals for these courses have included:

- mastering the elementary modeling and data analysis techniques presented in the text well enough to solve exercises like the ones at the ends of each chapter individually *and* to carry out the larger-scale chapter projects in groups of three or four several times over the course of a semester,
- understanding the underlying science well enough to appreciate the context of those problems and, finally,
- developing skill at presenting the results of quantitative investigations orally and in writing.

I have had the luxury of teaching two-semester sequences of courses in our first-year program so I have covered all of this material (together with other readings, writing assignments based on other sources, etc.) over the course of two semesters. As a result, there is more material here than could be comfortably covered in a one-semester course; in almost all cases, some choices will be necessary for courses taught in that format.

For classes with students who have a strong mathematical and scientific backgrounds from their high school coursework, the basic material on systems of measurement, linear and logarithmic scales, ratios, proportions, percents, etc., from Chapters 1 and 2 could be omitted or covered very quickly. The extended Part I Summary Project from Chapter 3 could also be used to review these concepts in the context of understanding characteristics of the U.S. energy economy and deriving understanding from information presented in a graphical and numerical format. Other classes may need a more substantial review of this material and it is very important to have these ideas well in hand for a number of later discussions.

[5] R Core Team, R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna Austria, 2015, https://www.R-project.org.

Various combinations of the chapters on modeling topics from Part II would be possible. For instance, the chapters on linear and exponential models from Chapters 4 and 5 are probably more important for later discussions than Chapter 6 on power law models. However, there are a number of interesting biological examples that lead to the consideration of power laws, so that material might also be appropriate for many courses. In any case, I would strongly recommend including topics from both the functional modeling sections in Chapters 4, 5, and 6 and the discussions of dynamic modeling via difference equations in Chapters 7 and 8.

Part III is not intended as a replacement for an elementary statistics textbook, but it does cover much of the same ground, so depending on the specific goals of different courses and the backgrounds of the students, more or less of that material might be appropriate.

In my thinking, the chapter projects are an integral part of this book and I would strongly recommend using as many of them as can fit comfortably in any course using this book. In some cases, you may want to do some preprocessing on the data sets to be used by the students. For example, if you use the Mauna Loa monthly atmospheric CO_2 concentration data for the Chapter 4 project, you will probably want to create a spreadsheet containing the most recent posted data. Similarly, the World Health Organization Ebola data set for the Chapter 5 project is presented in an unhelpful format on the Wikipedia page mentioned there; your students will thank you if you put that in a more usable format. If you really want to address the difficulty of curating and cleaning real-world data, however, you may want to turn your students loose without this sort of help.

I have typically taken one or two class days on each project and had students work in groups of three or four in a sort of lab format during class. Most of these projects are structured around computations in spreadsheets or the `R` statistical package. Virtually any spreadsheet software is sufficient here. My students have successfully used Excel, Google Sheets, and LibreOffice Calc. `R` is freely available and though it rightly has the reputation of possessing a steep learning curve, I have found that it is manageable with guidance from an experienced user. Familiarity and skill in using both of these sorts of software is valuable for students who will go on to major in a wide range of quantitative subjects. At my home institution, almost all students have laptop computers and the software needed is either already available or easily downloadable. So we have even used our regular classroom as the venue for these sessions. But computer labs would also work well, as long as students are able to convene in groups with working space on some kind of horizontal surface in addition to space for the computers they are using.

Comments and Corrections

I welcome comments, suggestions, and criticism from readers. Although I have made serious efforts to ensure the accuracy of the text, I know there are things I may have gotten wrong or misrepresented. So I would be glad to receive corrections for those. I would also be most interested to hear about how instructors have used this book in their own courses. Please send any such communications to

`jlittle@holycross.edu`.

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