AMS PAGE A DAY CALENDAR

The *AMS Page a Day Calendar* is a collection of 366 mathematical morsels. Each day features a fun math fact, a tidbit of math history, a piece of art made using mathematics, a mathematical puzzle or activity, or another mathematical delight. Topics range from the serious to the silly, from the abstract to the very real. The calendar features mathematics done by people from different races, genders, geographic locations, and time periods. Anyone interested in mathematics will learn something new and have their imagination sparked by something they find in the calendar. It will be a mathematical companion for your year.

If your life is ergodic, and a lot of the time it is, you'll keep bumping into certain things more often than others. What are those things you'll bump into more often? Well, the things that have higher measure for you, have higher meaning.

—Ami Radunskaya

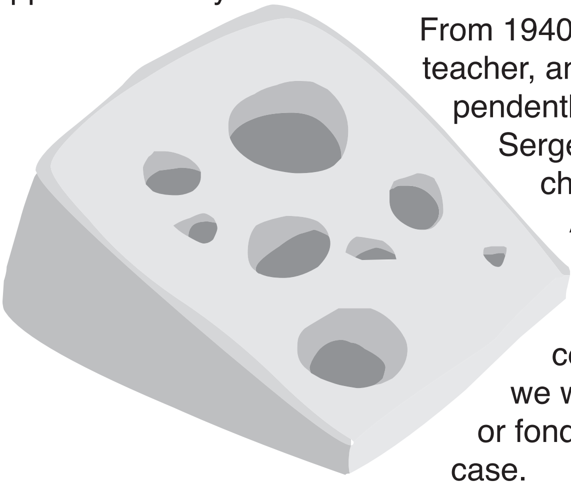
A function from a space to itself is said to be *ergodic* if the only invariant sets—that is, the only sets that are mapped to themselves—have either measure 0 or the full measure of the space. The Birkhoff ergodic theorem states that the following characterization is equivalent: a function is ergodic if the average behavior of a generic point in the space over repeated iterations of the function is the same thing as the average behavior the map over the whole space during one iteration.

Today is a good day to reflect on the ergodicity of life and whether you are giving the things that have the highest measure for you the most time in your life. Happy New Year!

Further information: Kevin Knudson and Evelyn Lamb, "Episode 9: Ami Radunskaya," *My Favorite Theorem* (podcast)

01 **JANUARY**

In mathematics, a "Swiss cheese" is a space formed by removing a countable union of open discs from a closed disc. Without further conditions on the removed discs, such a set might not be very interesting. But by adding the condition that the removed discs must have disjoint closures and that the interior of the Swiss cheese is empty, we end up with a version first cooked up in 1938 by Swiss mathematician Alice Roth to give an example of a compact space on which not every continuous function could be approximated by rational functions.



From 1940-1971, Roth worked as a secondary school teacher, and her example was forgotten. It was independently developed by Armenian mathematician Sergey Mergelyan in 1952 and nicknamed "Swiss cheese" by someone unfamiliar with Roth's work. After her retirement, she began working actively in mathematics research again.

We're pretty sure Swiss Cheese Day, celebrated January 2, is not a real holiday. But we will not advise against having some Emmentaler or fondue as you ponder approximation theory, just in case.

Further information: Ulrich Daepp, Paul Gauthier, Pamela Gorkin, and Gerald Schmieder, "Alice in Switzerland: The Life and Mathematics of Alice Roth," *The Mathematical Intelligencer* (2005)

02 JANUARY

American Mathematical Society

On this day in 2018, the Great Internet Mersenne Prime Search announced that the 50th Mersenne prime had been found. As with all Mersenne primes, this one has the form 2^p-1 where *p* is itself prime. (In this case, *p*=77,232,917.) This new discovery exceeded its predecessor by nearly a million digits and was found by a computer belonging to Tennessee electrical engineer Jonathan Pace, who had donated computer time to the search for Mersenne primes for 14 years before the discovery was made.

One reason people look for Mersenne primes is the Lucas-Lehmer test, a primality test for Mersenne numbers. First we construct the recursive sequence shown to the right, where the first term is 4 and each new term is 2 less than the square of the previous term. The number 2^n-1 is prime if and only if s_{n-2} has a remainder of 0 when divided by 2^n-1. Try it yourself for a few numbers. We suggest *n*=11 to start.

$s_0=4$
$s_1=14$
$s_2=194$
.
.
.
$s_i=(s_{i-1})^2-2$

The Lucas-Lehmer test is computationally cheap, which makes it easier to check Mersenne numbers for primality than other numbers of the same magnitude. But our luck may run out; no one knows whether there are infinitely many Mersenne primes. Each gem we dig up could be our last.

Further information: *Great Internet Mersenne Prime Search*, www.mersenne.org
J.W. Bruce, “A Really Trivial Proof of the Lucas-Lehmer Test,” *American Mathematical Monthly* (1993)

03 JANUARY