

Index

- $(a.b.c\dots)$, 90
- C_n , 41
- D_n , 42
- D_∞ , 45
- $F_{\mathcal{L}}$, 32
- $G_{\vec{v},p}$, 32
- $H(T)$, 113
- $R_{\theta,p}$, 31
- $S(Q)$, 41
- $S(\mathcal{T})$, 47
- $T_{\vec{v}}$, 31
- $[n_1.n_2\dots n_r]$, 96
- \mathring{A} , 14
- $\bar{e}(\mathcal{T})$, 148
- $\bar{v}(\mathcal{T})$, 148
- ∂A , 16
- σ -morphic protoset, 82
- $\{n\}$, 87
- j -valent tiling, 25
- k -isohedral tiling, 51
- k -uniform tiling, 91
- n -omino, 80
- n_1, n_2, \dots, n_r , 88
- $n_1.n_2\dots n_r$, 88
- $orb_H(p)$, 48
- r -morphic protoset, 75
- 1-skeleton, 246
- 2-manifold, 255
- 3-manifold, 248, 255
- 3-torus, 255

- Abelian group, 17
- adjacent tiles, 25
- affine transformation, 59
- anisohedral, 52, 154, 235

- antipodal map, 215
- antipodal point, 215
- antiprism, 216
- aperiodic protoset, 168
- Archimedean solids, 216
- Archimedean tiling, 90
- Archimedes, 217
- arm, 181
- aspect, 99, 122, 129, 173, 239, 249
- average number of edges per tile, 148
- average number of vertices per tile, 148

- Bailey, Duane, 200
- balanced tiling, 163
- Berger, Robert, 168
- Bierberbach, Ludwig, 59
- bijection, 8
- bijective function, 8
- Bolzano-Weierstrauss Theorem, 134
- boundary of a set, 16
- bounded set, 15
- Bourke, Paul, 123
- brickwork, 102

- Cairo tiling, 156
- capsid, 213
- casting, 243
- Catalan tilings, 272
- centroid, 136
- circle group, 46
- closed set, 14
- commute, elements in a group, 17
- compact, 15
- complement, 7
- composition of a tiling, 174
- composition of isometries, 12

- congruent subsets, 12
- congruent tilings, 28
- conjugation, 18
- convergence, 13
- convergence of sequence of tiles, 135
- convex, 12
- Conway Criterion, 77, 83, 266, 272, 278
- Conway’s Magic Pen, 266, 278
- Conway, John, 70, 71, 77, 130, 173, 239
- Coombs, Debora, 200
- cornered tile, 180
- cornerless tile, 180
- corona, 112, 276
- countable, 9
- countable tiles in a tiling, 27
- covering, 21
- cross, 181
- crystallographic groups, 59
- Crystallographic Restriction, 66
- cube-with-knotted-holes, 242
- CurvedSpaces, 278
- cyclic group of order n , 41

- Danzer, Ludwig, 236, 239
- Dawson, Robert, 218
- de Bruijn, Nicolaas Govert, 195, 199
- Decidability Problem, 168
- Delaunay, Boris, 237
- dense set, 27
- dense subset, 14
- diagonalization argument, 10, 93
- diameter, 49, 108, 141
- diameter of a tile, 25
- dihedral angle, 258
- dihedral group of order $2n$, 42
- dihedral group of uncountable infinite order, 45
- dihedral tiling, 23
- dimorphic, 241
- discrete set of isometries, 48
- domino tiling, 124
- domino tilings, 102

- edge of a tiling, 23
- einstein, 170, 209, 229, 240
- elements of a set, 7
- elliptic isometry, 225
- enantiomorphic pair, 78, 90
- Engel, P., 236
- enveloping a tile, 115
- epsilon-disk, 13
- equitransitive tiling, 103
- equivalence relation, 7
- equivalent tiles, 51
- equivalent tilings, 28
- Euclidean metric, 11
- Euclidean plane, 11
- Euler characteristic, 144, 260
- Euler, Leonhard, 144
- Extension theorem, 133

- face, 234
- face-to-face tiling, 234
- Fathauer, Robert, 276
- Federation Square, 174
- Fedorov, Evgraf, 59
- Fibonacci sequence, 128, 195
- flag, 55
- flag-transitive tiling, 55
- Fontaine, Anne, 80
- frieze group, 61
- fundamental domain, 252, 256

- genus two surface, 253
- geodesic, 11
- Girih, 263
- glide reflection, 32
- glide reflection, hyperbolic, 226
- golden ratio, 187, 195, 197



- Gomory's Theorem, 127
 Goodman-Strauss, Chaim, 193, 229
 Grünbaum, Branko, 236
 Greek Cross, 78
 group, 17
 group diagram, 59
 group, generating set, 17
 gyrobifastigium, 237, 239

 Hales, Thomas, 156, 238
 half-turn manifold, 256
 Hamiltonian cycle, 127
 handlebody, 242, 248
 Hantzsche-Wendt manifold, 256
 Hausdorff distance, 135
 Heegaard splitting, 248
 Heesch number, 113, 124, 229
 Heesch, Heinrich, 53
 Hilbert's Problems, 52, 59
 Hilbert, David, 52, 59
 homeomorphism, 15
 Honeycomb Conjecture, 156, 238
 hyperbolic isometry, 225
 hyperbolic plane, 222
 hyperbolic plane tiling, 254
 hyperbolic triangle, area, 224
 hyperplane, 271

 icosahedrite, 170
 ideal triangle, 224, 229
 incident, 25
 index set, 7
 inflation, 171
 injection, 8
 injective function, 8
 inradius, 137, 160, 176
 integers, 7
 interior of a set, 14
 isogonal, 97, 103
 isogonal tiling, 54, 271
 isohedral, 97
 isohedral tiling, 51
 isometric, 12
 isometry, 12
 isometry determined by action on three
 noncollinear points, 33
 isometry is product of up to three
 reflections, 34
 isometry preserves angles, 33
 isometry preserves lines, 33
 isometry, direct, 31
 isometry, identity, 31
 isometry, indirect, 31
 isometry, orientation-preserving, 31
 isometry, orientation-reversing, 31
 isometry, trivial, 31
 isomorphic group diagrams, 60
 isomorphism, 18
 isotoxal tiling, 54, 271
 IUCr notation, 70

 Johnson, Norman, 218
 Jordan Curve Theorem, 26

 kaleidoscope tiling, 218, 220
 Kaleidotile, 277
 Kari, Jarkko, 169
 Kelvin's Conjecture, 238
 Kelvin, Lord, 238
 Kepler, Johannes, 67, 90, 272
 Kerschner, Richard, 154
 kites and darts, 187
 Klein bottle, 253

 Lagrange's Theorem, 17
 lattice, 65
 Laves tiling, 96, 156
 Laves, Fritz, 96
 least perimeter, 156



- level one triangle, 206
- level zero triangle, 205
- limit point, 15
- linear transformation, 13
- longitude, 251
- lozenge tiling, 129
- lunar tiling, 218
- lune, 213

- Mann, Casey, 106, 114, 116
- Martin, George, 80
- matching condition, 187
- meridian, 251
- Moöbius band, 253
- monogonal, 90
- monogonal tiling, 55
- monohedral tiling, 23
- monomorphic, 241
- monomorphic protoset, 75
- Morgan, Frank, 156
- Mrs. Perkins' Quilt, 272
- Myers, Joseph, 80

- neighbor, 250
- neighborhood, 116
- neighborhood of a tile, 25
- neighboring tiles, 25
- non-edge-to-edge tilings, 102
- non-translational tiling, 168
- nonorientable 3-manifold, 257
- nonorientable surface, 253

- open set, 14
- orbifold notation, 70
- orbit, 48
- order of a group, 17
- outradius, 137, 141, 142, 151, 160

- packing, 21
- parallelepiped, 234, 236
- partition, 7
- patch, 28, 109, 116
- patch engulfed by a patch, 109
- patch extends to a larger patch, 109
- patch extends to a tiling of the plane, 109
- Pautze, Stefan, 176
- Penrose, Roger, 170
- pentagrid, 270
- period parallelogram, 65
- periodic, 263
- periodic tiling, 63, 142
- Phelan, Weaire, 238
- Philadelphia, 8
- Pigeonhole Principle, 8, 27, 50, 136, 143
- pinwheel tiling, 173
- planigon, 272
- Platonic solids, 240
- Poincaré ball model, 258
- Poincaré disk model, 222, 258
- polygon, 16
- polygon, convex, 25
- polygon, corners of, 23
- polygon, regular, 16, 23
- polygon, sides of, 23
- polyhedral tiling, 235
- polyhedron, 234
- polyomino, 80
- prism, 216
- prismatic tiling, 156
- product of isometries, 12
- protoset, 22
- protoset admits a tiling, 22
- prototile, 22

- quaquaversal tile, 239
- quarter-turn manifold, 256
- quasicrystal, 232
- quasicrystals, 167



- Radin, Charles, 173, 239
 random tilings, 122
 reflection, 32
 regular n -gon, 87
 regular tiling, 55
 regular tiling of the hyperbolic plane, 227
 regular tilings of sphere, 216
 regular vertex, 96
 Reinhardt, K., 53, 154
 relation, 7
 reptile, 171
 rigid transformation, 12
 Robinson's aperiodic protoset, 180
 Robinson, Raphael, 170, 180
 rotation, 31
 rotationally symmetric arc, 265
 rotationally symmetric arc, 76, 77

 SCD tile, 239
 Schattschneider, Doris, 106
 Schechtman, Dan, 167
 Schläfli notation, 56, 215
 Schmitt, Peter, 82, 239
 Schmitt-Conway-Danzer tile, 239
 screw motion, 233
 Seifert-Weber 3-manifold, 259
 semiregular tiling, 90
 Senechal, Marjorie, 193, 275
 Shephard, G.C., 236
 Shubnikov-laves tilings, 272
 similarity tiling, 175
 sixth-turn manifold, 256
 smooth patch, 112
 Socolar, Joshua, 203
 solid torus, 242
 spandrel, 123, 124
 species of a vertex, 88
 spherical isometry, 215
 spherical tilings, 213
 squared square problem, 106
 squared square, perfect, 106
 stabilizer subgroup, 51
 staircase, 195
 standard embedding, 242
 stereohedron, 236, 272
 strongly aperiodic, 240
 Sturmian word, 201
 subgroup, 17
 substitution tiling, 173
 substitution tilings, 278
 surjection, 8
 surjective function, 8
 surround number, 115
 symmetry group, 41
 symmetry of a shape, 41
 symmetry of a tiling, 47
 symmetry type, 60

 Tasarov, A.S., 229
 Taylor, Joan, 203
 tessellation, 22
 tetrahedral tiler, 237
 third-turn manifold, 256
 Thurtson, W.P., 130
 tic-tac-toe, 278
 tile, definition of, 22
 tile-transitive tiling, 51
 tiling not edge-to-edge, 102
 tiling over a disk, 133
 tiling, n -hedral, 23
 tiling, definition of, 22
 tiling, edge-to-edge, 24
 tiling, regular, 23
 topologically equivalent, 15
 torus, 66
 torus earth, 251
 transitivity class of tiles, 51
 translation, 31



- triangle inequality, 11, 33
- triangulation, 246
- trilobite and cross, 193
- Truchet tilings, 122
- Truchet, Sébastien, 122
- Turing machine, 170, 269
- type of a vertex, 88

- uncountability of the real numbers, 10
- uncountable, 10
- uniform, 103
- uniform tiling, 90
- unilateral tiling, 105
- upper-half-plane model, 230, 270

- Valence Restriction Equation, 96

- valency of a vertex, 25
- vertex, 234
- vertex of a tiling, 23
- Vertex Restriction Equation, 87
- Voronoi tiling, 232

- wallpaper groups, 59, 67
- Wang tile, 168
- Wang, Hao, 168
- Weaire, Denis, 238
- Weaire-Phelan tiling, 238
- weakly aperiodic, 240

- X-ray diffraction, 167

- Zalgaller, Viktor, 218

