Preface

This book is about the mathematics behind card shuffling. Which shuffling methods are the most effective? What happens after repeated shuffles? How can such knowledge be used to improve one's toolset in card games (bridge, poker, etc.) or to understand card tricks? These questions can be investigated using a variety of mathematical tools, from the elementary (such as the notion of "carries" in arithmetic) to the more sophisticated (such as Markov processes and other notions from probability theory). We note that Markov processes are a widely used tool in scientific computing (for physics, statistics, biology, the social sciences, business, and the military). Having real examples (such as ordinary riffle shuffles) with sharp, definitive answers gives insights into Markov processes.

In writing this book, we were guided by the desire to stay rooted in ideas that come from real-life situations, while at the same time exploring connections with a wide range of mathematical fields. Depending on the reader's background, some chapters will be less easily accessible than others, but throughout the book, we will continually bring things back to hands-on experiments that can be done by real people shuffling real cards. There are a slew of good card tricks based on the imperfections of shuffling (see Chapter 16). Crooked gamblers have studied shuffling, both fair and foul, since the invention of playing cards. Many of their insights illuminate important mathematical concepts.

A secondary theme of our book is to share some of the beauty and power of modern mathematics, using shuffling as a touchstone from which to branch out and return. As the reader will see, shuffling cards involves probability, analysis, combinatorics, and algebra. The mathematical developments that start with shuffling also touch geometry, topology, and number theory. The flow of ideas between shuffling and mathematics goes both ways.

Who are we and who is the intended audience? This book is a report of two lifetimes' work on the mathematics of shuffling cards. Persi Diaconis is a professor at Stanford University and began the study at age twelve or so through card tricks. This would lead to gambling and finally to mathematics. Jason Fulman is a professor at the University of Southern California and wrote a Ph.D. thesis with Diaconis on probability and group theory. He has been hard at work bringing modern mathematics, particularly combinatorics and Lie theory, to advance the understanding of shuffling. Both authors have integrated card shuffling into their teaching and hope that this book will be a useful resource for other instructors in mathematics.

This book is aimed at advanced undergraduate math majors, math graduate students, and postdoctoral researchers in mathematics who have a solid background

in probability, combinatorics, and group theory. Parts of the book address the history of shuffling and magic tricks and may be of interest to a broader audience. The introduction gives a synopsis of the chapters and references to needed mathematical background.

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