

Index

- 0-divisor, 132
- G -set, 99, 121
- p -group, 120, 121
- p -primary subgroup, 104, 154

- ABC Conjecture, 156
 - Mason's Theorem, 160
 - Polynomial ABC, 160
- affine multi-linear, 109, 188, 189
- anti-symmetric relation, 5
- arithmetic progression, 172
- Artin, Emil, xv
- associative property, 2, 53

- ball
 - closed, 51
 - open, 51
- Bayes's Theorem, 250
- Bell number, 224
- bijective, 10
- binary
 - operation, 94
 - sequence, 235
- binary rooted tree, 238
- binomial
 - coefficients, 214
 - Taylor expansion, 268
 - theorem, 181

- Catalan
 - equation, 168
 - numbers, 236
 - sequence, 237
 - walks, 236
- Cauchy's Theorem, 120
- Cauchy-Schwarz Inequality, 57
- centralizer, 101
- characteristic
 - function, 129, 183
 - of a ring, 132
 - polynomial, 298
- Chinese Remainder Theorem, 134, 179
- class equation, 101, 120
- closed interval, 3

- closure, 53
- comaximal ideals, 134
- commensurable, 22, 60
- commutative, 53
 - group, 53
 - ring, 129
- compact, 70
- complement, 2
- complete relation, 5
- Complex numbers, 280
 - conjugate, 280
- complex polynomial dynamics, 197
- congruence
 - mod a subgroup, 103
 - mod an ideal, 131
 - modular, 33, 34
- conjugacy class, 101, 102
- conjugate
 - complex, 280
 - quaternion, 297
- connected, 73
- continuous, 53
- contrapositive, 7
- converse, 7
- convex hull, 74
- cryptology, 84
- cubes
 - sums of consecutive cubes, 271

- Dehn, Max, 328
- dense, 53
- derangement, 185
- digits
 - base- b , 19
 - significant, 27
- dihedral group, 123
- Dirichlet Theorem on Arithmetic Progressions, 172
- discrete, 52
 - calculus, 263
 - discrete dense dichotomy, 58
 - probability, 243
 - uniformly discrete, 52
- disjoint sets, 2

- distributive, 2, 129, 180
- division with remainder (DwR), 18
 - as a function of a and b , 19
 - in school mathematics, 19
- dodecahedron, 114–116, 119
- dot product, 56, 57
- dual polyhedra, 115
- dynamics
 - complex polynomial, 197
 - integer polynomial, 195
- edge, 113
- empty set, \emptyset , 2
- equivalence relation, 5, 6, 34
- Erdős-Wood conjecture, 158
- Euclidean
 - n -space, 56
 - algorithm, 20, 22, 63
 - distance, 72
 - infinitude of the primes, 81
 - isometry groups, 109
 - norm, 56
 - plane, 51
 - tiling, 21, 23
 - transformations, 290
- Euler's
 - formula, 113
 - phi-function, 106, 261
 - theorem, 219
- even
 - functions, 210
 - number, 2
- exponential
 - function, 283
- exponentials, 15
- face, 113
- factorization, 81
 - prime power, 81, 83, 88, 141
 - principal ideal domain (PID), 151
 - unique, 145, 148
 - unique factorization domain (UFD), 150, 151
- Fermat
 - 's Last Theorem, 137, 157, 290
 - numbers, 90
 - primes, 90
- Fermat's Last Theorem, 164
- field, 130
 - of fractions, 131
 - of fractions of an integral domain, 130
- finite
 - cyclic groups, 105
 - isometry groups of \mathbb{R}^3 , 113
- fixed point, 99
 - set of, $\text{Fix}_G(X)$, 99
- Four-square Theorem, 302
- fraction
 - field of fractions, 130
 - fractional part of a , $\langle a \rangle$, 20
 - integer part of a , $[a]$, 20
- free action, 100
- Frobenius
 - endomorphism, 216, 217
 - numbers, 67, 68
- function
 - ring of symmetric functions, 186
- functions, 10
 - bijjective, 10
 - continuous, 53
 - even, 210
 - graph, 10
 - injective, 10
 - isometric, 53
 - odd, 210
 - surjective, 10
- Fundamental Theorem
 - of Algebra, 179, 288
 - of Arithmetic, 80
 - of Discrete Calculus, 231, 266
- geometric series, 15
- graph
 - combinatorial, 261
 - of a function, 10
 - planar, 219
- greatest common divisor, 7, 17, 20, 60
- group, 96
 - G -set, 99
 - action, 99, 100, 186
 - additive group, 53
 - alternating group, 119, 144
 - automorphism, 98
 - center, 98, 101
 - centralizer, 101
 - commutative, 53
 - commutative group, 83, 103, 123, 128, 138
 - cyclic group, 54, 58, 97, 105, 286
 - dihedral group, 97, 116, 123
 - elements of finite order, 103
 - finite, 101, 113, 120, 121
 - free action, 100
 - generators, 97
 - group of automorphisms, $\text{Aut}(A)$, 98
 - homomorphism, 65, 97
 - isomorphism, 98
 - multiplicative, 65, 83, 135, 285
 - normal subgroup, 96
 - normalizer, 102
 - of quaternions, 298
 - orbit, 99
 - permutation group, 97
 - power map, 103
 - semi-group, 96
 - stabilizer, 99
 - subgroup, 54, 96

- Sylow Theorems, 120, 121
- symmetric group, 97, 119, 144, 186, 189, 221
- torsion, 103
- transitive action, 99
- group action, 123
- groups
 - p -groups, 121
- half-line, 51
- homomorphism, 65, 97
 - conjugation, 98
 - inner automorphisms, 98
 - kernel, 97
 - ring, 105, 131
- Hurwitz quaternions, 300, 301
- icosahedron, 114
- ideal, 131, 301
 - comaximal, 134
 - left, 301
 - nil, 133
 - prime, 133
 - principal, 151
 - right, 301
 - Waring Theorem, 171
- idempotent, 132
- identity element, 10, 11, 53, 95
 - additive, 53
 - multiplicative, 96
- incidence algebra, 228
- Inclusion-Exclusion formula, 183, 184
- index of a subgroup, $[G : H]$, 101
- induction, 9
- injective, 10
- inner automorphism, InAut, 98
- inner product, 57
- integer
 - multiple, 35
 - multiples, $\mathbb{Z}b$, 34
- integer part of a : $[a]$, 20
- integer polynomial dynamics, 195
- integers
 - \mathbb{Z} , 6
 - mod m , 132
- integral domain, 129
 - principal ideal domain, 151
 - principal ideal domain (PID), 151, 153
 - set of representatives, 153
 - unique factorization domain (UDF), 153
 - unique factorization domain (UFD), 151
- Intermediate Value Theorem, 74
- interpolation (polynomial), 179
- intersection, 2
- interval
 - closed, 3, 51
 - half-open, 51
 - open, 51
- inverse
 - additive, 53
 - element, 53, 95
 - function, 10
- irreducible, 147
 - element, 147, 153
 - polynomial, 155, 289
- isolated points, 52
- isometric, 53
- isometry groups, $\mathbb{E}(n)$, 109, 113
- isomorphism, 98, 102
 - group, 98, 102
 - ring, 133
- Jacobson radical, 173
- joins, 218
- kernel, 97
- Klein, Felix, xii
- Lagrange
 - Four-square Theorem, 302
 - Lagrange's Theorem, 101
- Lang, Serge, xv
- least common multiple, 7, 61
- Mason's Theorem, 160
- Mersenne
 - numbers, 90
 - primes, 90
- metric space, 50
 - bounded, 51
 - Cartesian product distance, 72
 - closed ball, 51
 - closure, 53
 - compact, 70
 - connected, 73
 - convex, 74
 - dense, 53
 - discrete, 52
 - Euclidean distance, 72
 - Euclidean metric, 51
 - metric, 50
 - open ball, 51
 - sphere, 51
 - Taxi cab geometry, 52
 - translation-invariant, 51
- Mobius inversion, 228
- modular
 - congruence, 34
 - rings, 135
- modular congruence, 33, 34
- Monty Hall problem, 253
- multi-linear, 189
- multinomial
 - coefficient, 220
 - theorem, 182
- multiple, 18, 34, 35
- multiples, 79
- multiplicative

- function, 90, 261
- group, 135, 285
- identity, 296
- natural numbers, \mathbb{N} , 6
- nil ideal, 133
- nilpotent
 - element, 132
 - ideal, 133, 158
- norm, 51, 55
 - Euclidean, 51, 56
 - Taxi cab, 52
- normal subgroup, 96, 97, 102
- normalizer, $N_G(H)$, 102
- octahedron, 114
- odd
 - functions, 210
- orbit, 12, 99
 - equation, 99
- order of a group/element, 103
- order of magnitude, 27
- orthogonal
 - complement, 59
 - group, 109, 290, 303
 - projection, 59
- parallelogram, 304
- partial fraction
 - decomposition, 154
 - integration of real rational functions, 155
- partial fractions, 153
- partial order, 5, 146, 228
- partially ordered set, 147
- partition, 5, 6, 99, 223, 248
 - Bell number, 224
 - Stirling number of the second kind, 223
- period
 - of a function, 195
 - of an orbit, 12
- periodic function, 76, 77
 - Set of all periods, $\text{Per}(f)$, 77
- permutation
 - derangement, 185
 - discernment problem set, 319
 - fixed points, 249
 - group, 97
 - sign, 144
- permutations, 11
- phi-function (Euler), 306
- pigeonhole principle, 13
- Pillai's conjecture, 168
- planar graph, 219
- Platonic solids, 113
- polyhedra
 - dual, 115
- polyhedron
 - convex, 113
 - regular, 113, 114
- polynomial, 176
 - binomial theorem, 180
 - constant term, 176
 - degree, 176
 - division with remainder, 177
 - identities, 15
 - in several variables, 144
 - interpolation, 179
 - leading coefficient, 176
 - leading term, 176
 - quartic, 205
 - quintic, 205
 - ring, 142, 144
 - root, 176
- poset, 228
- power set, 2, 213
- prime
 - valued polynomials, 83
 - element, 133, 147
 - factorization, 81, 83
 - ideal, 133
 - number, 7, 79
- primitive root mod p , 107
- principal ideal domain (PID), 153
- projection, 100
- proof by contradiction, 7
- proof by induction, 9
- Pythagorean
 - theorem, 290, 291
 - triple, 290, 292
 - triples, 294
- quadratic reciprocity, 138
- quadrilateral, 304, 310
- quantifier, 8
 - existential, \exists , 8
 - universal, \forall , 8
- quartic polynomial, 205
- quaternion
 - group, 298
- quaternions, 296
 - conjugation, 297
 - Hurwitz quaternions, 300
 - integer quaternions, 299
 - norm, 297
 - pure, 298
 - reduced norm, 297
 - set of coefficients, 299
 - trace, 297
- quintic polynomial, 205
- quotient, 6, 100
 - group, 100, 102, 103
 - ring, 131
 - set, 102
- radical, 157
 - in a UFD, 159
- rational multiple, 59
- rational number, \mathbb{Q}

- base- b representation, 40
- rational numbers, \mathbb{Q} , 6
- real additive group, 49, 58
- real numbers, \mathbb{R} , 6
- reflections, 97, 116
 - hyperplane, 109
 - plane reflection, 109
- reflexive property, 146
- reflexive relation, 5
- relations, 5
- remainder, 18
 - $r_m(a)$, 19, 37
- ring
 - 0-divisor, 132
 - characteristic, 132
 - commutative, 129
 - examples, 129
 - field, 130
 - finite integral domain, 133
 - homomorphism, 105, 131
 - ideal, 131
 - idempotent, 132
 - idempotent element, 132
 - integral domain, 129
 - nilpotent, 132
 - nilpotent element, 132
 - prime element, 133
 - prime ideal, 133
 - quotient, 131
 - quotient ring, 131
 - unit, 132
 - zero-divisor, 132
- root
 - n th roots in \mathbb{C} , 287
 - complex roots, 205, 290
 - of a polynomial, 176, 205
 - primitive, mod p , 107
 - square, cube root, 205
- rooted tree, 238
- RSA cryptography, 87
- rules of arithmetic, 6, 94, 129, 130, 310
- semi-group, 96
- sequence
 - arithmetic progression, 172
 - as a function, 264, 265
 - binary, 235
 - convergent, 52
 - Fibonacci, 63
 - in the Euclidean Algorithm, 22, 23
- sets, 1
 - cardinal, 13
 - cartesian product, 3
 - complement, 2
 - empty set, 2
 - intersection, 2
 - relations, 2, 5
 - union, 2
- Simpson's Paradox, 252
- special orthogonal group, $\text{SO}(n, \mathbb{R})$, 109
- Sperner's Theorem, 234
- sphere, 51
 - unit sphere, 298
- Stirling numbers
 - of the first kind, 226
 - of the second kind, 223
- sub-
 - group, 54
 - ring, 129
 - set, 2
- Sum of consecutive squares, 272
- surjective, 10
- Sylow Theorems, 120, 121
- symmetric
 - functions, 186, 187
 - group, 97, 119
 - relation, 5
- symmetry, 50, 51
 - absolute, 61
 - and multi-linearity, 188
 - in Platonic solids, 113, 115
- Tate, John, xv
- Taxi-cab geometry, 52
- tetrahedron, 114
- torsion, 103
 - free, 104
 - subgroup, 103, 104
- transitive
 - group action, 99, 120, 121
 - property, 146
 - relation, 5
- translation, 63
 - group of, 108
 - invariant, 53, 55
- transposition, 145
- trapezoid, 320, 321
- triangle inequality, 57
- union, 2
- unique factorization domain (UFD), 150, 295
- unit, 96, 132
 - multiplicative group of units, 105, 135
- vertex, 113
- whole number, 19, 156–158
- Wieferich primes, 137, 166
- zero divisor, 132