
Contents

Preface	xi
List of Commonly Used Symbols	xv
Chapter 1. Introduction	1
1.1. The Theorems of Poncelet and Cayley	1
1.2. The Poncelet and Steiner Theorems— A Misleading Analogy	6
1.3. The Real Case of Poncelet's Theorem	9
1.4. Related Topics	10
Part I. Projective Geometry	
Chapter 2. Basic Notions of Projective Geometry	15
2.1. Projective Plane	15
2.2. Projectivities	19
2.3. Projective Line	22
2.4. Algebraic Curves	24

Chapter 3. Conics	31
3.1. Conics	31
3.2. Intersection of Line and Conic	34
3.3. Reduced Form	36
3.4. Projective Structure on a Smooth Conic	38
3.5. Parametric Equations of Smooth Conics	39
Chapter 4. Intersection of Two Conics	43
4.1. Intersection Numbers	43
4.2. Bezout's Theorem for Conics	51
4.3. Conic Pencils	53
4.4. Degenerate Conics in a Conic Pencil	55
Part II. Complex Analysis	
Chapter 5. Riemann Surfaces	61
5.1. Definition of Riemann Surface	61
5.2. Examples of Riemann Surfaces	65
5.3. More Examples of Riemann Surfaces. Algebraic Curves	68
5.4. Examples of Conformal Maps	74
5.5. Covering Surfaces	76
5.6. Isomorphisms of Tori	79
Chapter 6. Elliptic Functions	83
6.1. Elliptic Functions	83
6.2. The Weierstrass \wp -Function	86
6.3. The Functions ζ and σ	89
6.4. Differential Equation for \wp	92
6.5. The Elliptic Function $w = sn(z)$	94
Chapter 7. The Modular Function	97
7.1. The Functions g_2, g_3	97
7.2. The Modular Function J	98
7.3. Fundamental Region for Γ	100

7.4. Fourier Expansion of J	102
7.5. Values of J	104
7.6. Solution to the Inversion Problem	108
Chapter 8. Elliptic Curves	111
8.1. Elliptic Curves	111
8.2. Algebraic Models	113
8.3. Division Points of C/Λ	115
8.4. Division Points of \mathcal{S}	117
 Part III. Poncelet and Cayley Theorems	
Chapter 9. Poncelet's Theorem	123
9.1. Poncelet Correspondence	123
9.2. Algebraic Equation for \mathcal{M}	125
9.3. Complex Structure on \mathcal{M}	128
9.4. \mathcal{M} is an Elliptic Curve	130
9.5. The Automorphisms σ, τ , and η	131
9.6. Proof of Poncelet's Theorem	132
Chapter 10. Cayley's Theorem	135
10.1. Origin of \mathcal{M}	135
10.2. Algebraic Equation for \mathcal{M}	136
10.3. Proof of Cayley's Theorem	138
Chapter 11. Non-generic Cases	141
11.1. Fixed Points of η	141
11.2. Equations for C, D , and \mathcal{M}	142
11.3. The Riemann Surface \mathcal{M}_0	144
11.4. Formulas for η	147
11.5. Poncelet's Theorem	148
11.6. Existence of Circumscribed n -Gons	150

Chapter 12. The Real Case of Poncelet's Theorem	153
12.1. Poncelet's Theorem for Two Circles	153
12.2. Poncelet's Theorem for Two Ellipses	155
12.3. Topological Conjugacy	157
Part IV. Related Topics	
Chapter 13. Billiards in an Ellipse	165
13.1. Billiards in an Ellipse. Caustics	165
13.2. The Map η_R	167
13.3. Description of \mathcal{M}_R	168
13.4. Invariant Measure. Rotation Number	170
13.5. Billiard Trajectories with the Same Caustic	172
13.6. Derivation of Invariant Measure	173
13.7. Proofs of Theorems 13.3 and 13.4	177
Chapter 14. Double Queues	179
14.1. The Two-Demands Model	180
14.2. Formulas	182
14.3. Riemann Surface	183
14.4. Automorphy Conditions	184
14.5. The Regions \mathcal{D}_z and \mathcal{D}_w	184
14.6. Analytic Continuation	186
Supplement	
Chapter 15. Billiards and the Poncelet Theorem	
S. TABACHNIKOV	191
15.1. Mathematical Billiards	191
15.2. Integrable Case	195
15.3. Poncelet Grid	198
15.4. Poncelet Theorem on Quadratic Surfaces	204
15.5. Outer Billiards in the Hyperbolic Plane	207
References	210

Appendices

Appendix A. Factorization of Homogeneous Polynomials	215
Appendix B. Degenerate Conics of a Conic Pencil. Proof of Theorem 4.9	219
Appendix C. Lifting Theorems	223
C.1. Homotopy	223
C.2. Lifting Theorems	224
Appendix D. Proof of Theorem 11.5	229
Appendix E. Billiards in an Ellipse. Proof of Theorem 13.1	233
References	237
Index	239