

# Contents

Preface	xi
Credits	xiii
Acknowledgments	xvii
Overture	1
The Ideal Mathematician (with Philip J. Davis)	5
Manifesto	13
Self-introduction	17
Chronology	21
Mathematics Has a Front and a Back	35
<b>Part 1. “Mostly for the right hand”</b>	<b>41</b>
Introduction	43
True Facts About Imaginary Objects	47
Mathematical Intuition (Poincaré, Polya, Dewey)	51
Summary	51
Mathematical Intuition	59
Polya	61
Mental Models	63
Mental Models Subject to Social Control	66
Dewey and Pragmatism	68
Acknowledgments	72
To Establish New Mathematics, We Use Our Mental Models And Build On Established Mathematics	73
Introduction	73
Established mathematics	75
Mathematicians’ proof vs. axiomatic proof	77
Mathematicians’ proof is semantic, not syntactic	78
Established mathematics is fallible	80
Published vs. private, rigorous vs. plausible	82
Established mathematics is not controversial	83
Acknowledgments	85

How Mathematicians Convince Each Other or “The Kingdom of Math is Within You”	89
A quote from Hardy	90
What some mathematicians say they are doing	91
The Materialist versus the Platonist: Changeux and Connes	95
<i>What, then, is a mathematicians’ proof?</i>	101
Relation between formal proof and mathematicians’ proof	102
Aristotle, Kant, and Locke	104
Is this mere Platonism?	104
Heron’s area theorem	105
Conclusions	108
Acknowledgments	108
Appendix	108
On the interdisciplinary study of mathematical practice, with a real live case study	115
Wings, not foundations!	125
1. What foundations?	125
2. Lived experience as “foundation”	128
Inner Vision, Outer Truth	131
Mathematical Practice as a Scientific Problem	137
Atiyah’s pleasant surprise	137
Does “existence” matter?	137
For a multi-disciplined study of mathematical practice	139
The basic problem	140
Timely or timeless?	142
Conclusion	144
Educational implications	144
Proving is Convincing and Explaining	147
I. What is proof?	147
II. Proof among professional mathematicians	147
III. Three meanings of “proof”	148
IV. Variation in proof standards	149
V. The four-color theorem	150
VI. Proof in our classrooms	153
VII. Coda	155
Fresh Breezes in the Philosophy of Mathematics	157
Foundations lost	157
Phil / m and phil / sci	158
Taking the test	160
Definition of mathematics	163
Introduction to “18 Unconventional Essays on the Nature of Mathematics”	167

<b>Part 2. “Mostly for the left hand”</b>	173
Introduction	175
Rhetoric and Mathematics (with Philip J. Davis)	177
Part 1: Mathematics as Rhetoric	178
Part 2: Rhetoric in Mathematics	182
Closure	188
Math Lingo vs. Plain English: Double Entendre	191
Independent Thinking	195
The “Origin” of Geometry	199
The Wedding	205
Mathematics and Ethics	207
Ethics for Mathematicians	213
Under-represented Then Over-represented: A Memoir of Jews in American Mathematics	217
Paul Cohen and Forcing in 1963	227
<b>Part 3. Selected book reviews</b>	233
Introduction	235
Review of <i>Not Exactly...In Praise of Vagueness</i> by Kees van Deemter	237
Review of <i>How Mathematicians Think</i> by William Byers	241
Review of <i>The Mathematician’s Brain</i> by David Ruelle	247
Review of <i>Perfect Rigor</i> by Masha Gessen	251
Review of <i>Letters to a Young Mathematician</i> by Ian Stewart	255
Review of <i>Number and Numbers</i> by Alain Badiou	257
<b>Part 4. About the Author</b>	263
An amusing elementary example	265
Annotated research bibliography	267
Poems	269
Curriculum Vitae	271
List of articles	273
Index	279