

# Introduction

*Paraphrasing Aryeh Dvoretzky:  
Like the sun in the sky,  
Lipman Bers cast a giant light.*

This volume is an informative tribute to Lipman (Lipa) Bers from admirers, students, and friends. It contains expository articles by colleagues on Lipa's research. These are bracketed by an edited version of Bers' previously unpublished autobiographical notes and a reprint of his article on the migration of European mathematicians to the U.S.

The year 2014 marks the hundredth anniversary of Lipman Bers' birth and twentieth anniversary of his death. Several of his students decided to appropriately commemorate these dates with an informal meeting, Bers 100, at the Graduate Center of the City University of New York (CUNY) on May 19, 2014, and to produce this book.

This volume begins with

- Lipa's unfinished and previously unpublished "Pages from a Memoir" with additional introductory notes and reflections by his children Ruth Bers Shapiro and Victor Bers. We have also included some photos depicting his professional and family life,

followed by

- A reprint of an article by L. Nirenberg that describes Bers' early work on partial differential equations and includes newly added personal remarks.
- A note by W. Abikoff and R. Sibner about Bers' early work on differential equations, his research on pseudo-analytic functions and fluid flows. This article, as do many others in the volume, contains personal reminiscences.

The main mathematical emphasis of the book, however, is on Bers' published results on moduli of Riemann surfaces and complex dynamics because this work has had a major impact on today's mathematical activities in complex analysis and low dimensional geometry and topology. This is reflected in the following articles in the volume.

- A review by S. Wolpert of the Ahlfors-Bers Measurable Riemann Mapping Theorem, with a new proof of this fundamental result in the study of moduli problems. The deformation theory of Kleinian groups introduced

by Bers and Lars V. Ahlfors, as well as much of the modern work on Teichmüller spaces, is based on this result.

- The Ahlfors-Bers creation of the Modern Theory of Kleinian Groups: *A small acorn grows to a mighty oak*, a historical appreciation by A. Marden.
- The Bers embedding and (some of) its ramifications by I. Kra and B. Maskit, describing what many consider the most important Bers contribution to moduli theory. Lipa's representation of Teichmüller space as a complex analytic manifold is what has made Teichmüller theory so useful and extended its boundaries; this representation, called the Bers embedding<sup>1</sup>, is presented in this article. It also contains a mild reference to the mutual admiration and generous accreditation to others by both Lars Ahlfors and Lipman Bers.
- F. Gardiner's and L. Keen's article: Lipman Bers: a Retrospective. The concept of a holomorphic motion, although he didn't call it that, occurs first in Bers' work. This idea gives a framework for studying various deformation problems, most particularly in holomorphic dynamics. Holomorphic motions and its application to some recent results are the subject of this article.
- A note by G. Riera and R. E. Rodríguez on the Weil-Petersson geometry of a family of Riemann Surfaces, where a description is given of the tangent space to Teichmüller space in terms of Beltrami differentials.
- H. Masur's contribution on the Legacy of work of Bers on the Mapping Class Group. This paper by Masur traces a line of research that originated with William Thurston's topological/geometric classification of diffeomorphisms of Riemann surfaces and was reformulated and expanded by Bers using Teichmüller spaces; it was further developed by Kra, who gave a qualitative description of a class of these diffeomorphisms, and finally by S. Dowdall, who quantified the description.
- A short note on Bers' decomposition theorem by A. Basmajian and P. Susskind containing a new proof of Bers' theorems establishing bounds on the lengths of the shortest non-boundary curve and the lengths of the shortest curves defining a pants decomposition of the surface in terms of the topology and the lengths of the boundary curves.
- A long historical scholarly account of the theory of moduli of Riemann surfaces, beginning with the historic work of Bernhard Riemann and ending with some of the contributions of Bers to the field by N. A'Campo, L. Ji and A. Papadopoulos. This article discusses the contributions of giants to the topological and geometric foundations for the study of moduli, as well as the fundamental work of Teichmüller. The contrast between the brilliant mathematician Teichmüller and the awful political animal Teichmüller enter into this account as it did in Lipa's work. As with all historical accounts, some of the points of view expressed here are subject to alternate interpretations.

We have also included two notes directly related to the talks on May 19, 2014:

- I. Kra's note on Dennis Sullivan's and Jeremy Kahn's reminiscences of Bers.

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<sup>1</sup>The name is probably due to Ahlfors.

- Yair Minsky’s article, based on lecture notes by C. Arettines, of his talk: Bers embeddings, skinning maps and hyperbolic geometry. A video recording of Minsky’s talk can be found at

<http://tinyurl.com/YMinskyvideo>

Next are several reprinted articles by and about Bers that are appropriate to our tribute:

- The short note by Bers in the Bulletin of the American Mathematical Society (AMS) whose ostensible purpose is to correct a mistake in an earlier proof, but in reality announces what is probably Lipa’s most important paper. It contains the key result discussed by Kra and Maskit in their paper in this volume. The full proofs of theorems that strengthened the ones Bers announced appear in one of his subsequent papers.
- A historical paper by Bers on the migration of mostly European mathematicians to the U.S. and their remarkably friendly reception by colleagues. This paper updates his autobiographical notes and complements other historical entries in the volume. Perhaps the most important reason for including it is that the paper was dedicated to his beloved and almost constant companion, his wife, Mary.
- The obituary article on Bers by H. Bass and I. Kra prepared for the National Academy of Sciences and published in the Proceedings of the American Philosophical Society. The latter version appears in this volume.
- Reprints of sections from the Notices of the AMS obituary article “Remembering Lipman Bers” containing contributions by C. Morawetz, C. Corillon and I. Kra, T. Weinstein, and J. Gilman.
- Reprint of the article “Lipman Bers, A Mathematical Mentor” with contributions by L. Keen, T. Klotz Milnor, L. Sibner, I. Kra, J. Gilman, and J. Dodziuk.

Lipa’s published work is not limited to research papers and the historical remarks on the migration of mathematicians reprinted in this volume. Of historical interest may be his book in Yiddish on “Arithmetic books in Yiddish.” In 1969, Bers wrote a “Calculus” text. The preface he wrote in it reveals a lot about how he thought about and taught mathematics.<sup>2</sup> “I hope this is a modern calculus book, since it is written by a modern working mathematician. Whenever possible, examples from recent scientific development are used. But no attempt is made to use artificially modern language; traditional notation and terminology are preserved, with only a few exceptions. . . . Intuitive reasoning is used and stressed throughout the book, not as a substitute for, but rather as a guide to, rigorous thinking.” A clear illustration is given in the book’s section on limits and continuity, where he manages to convey the geometric meaning of the concepts, and work with them, long before the rigorous definition is given, at which point it almost seems natural.

In 1986 Bers gave an “extemporaneous” lecture in the Sullivan seminar at the Graduate Center of CUNY, which was quintessential Bers. This lecture, titled *My life with quasiconformal mappings*, was taped and is available at

<http://tinyurl.com/Bersvideo>

Sullivan asked Bers to speak because the scheduled lecturer Ricardo Mañé had to cancel due to illness. The lecture deals with Lipa’s research transition from applied

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<sup>2</sup>See J. Kahn’s remarks in the note by Kra.

mathematics to quasiconformal mappings and Teichmüller theory. He describes his search for an a priori estimate for quasiconformal self mappings of the unit disc and his success in finding a beautiful and useful one. The lecture, which demonstrates Bers' charm and sense of humor, is a tribute to the field of quasiconformal mappings and shows his admiration for its researchers' ethical behavior. It gives an excellent sense of Bers' mathematical generosity and general *Menschlichkeit*.

The Mathematics Genealogy Project credits Bers with 53 students, 16 of them women, and lists 368 descendants. In the best of socialist/humanist traditions, Lipa gave each of his students the advice and guidance he thought s/he needed, took pride in their diverse accomplishments and contributions and somehow inspired them to replicate the model with their own students.

The mark Bers left on analysis, geometry and topology is only partially reflected by the talks at the Bers 100 event and the mathematical content of this volume. Bers' humane and activist traditions come through in the autobiographical notes edited by his children and in some of the other biographical notes included here. Although he had many interests that included politics, human rights and civil liberties, Bers was above all a mathematician.

Lars Ahlfors and Lipman Bers were close collaborators and they created a close mathematical family consisting mostly of their students and sometimes called *The Ahlfors-Bers Mafia*. Lipa expanded the Mafia by bestowing honorary membership to "outsiders" such as Daniel Mostow, Dennis Sullivan and William Thurston, men whom he both admired and influenced greatly. In general, he was extremely supportive of "newcomers" to the field who took it in new and sometimes unexpected directions.

Their legacy is reflected in, among others, the tri-annual Ahlfors Bers Colloquia that started at CUNY in 1995<sup>3</sup> and its latest meeting in October 2014 at Yale, and the periodic Iberoamerican Congresses on Geometry since 1998, the sixth of which met at CUNY directly after Bers 100. The Colloquia and Congresses are substantially different in scope from the Bers 100 meeting; they deal with latest developments and new directions in complex analysis, Teichmüller theory, low dimensional geometry and topology. The Bers 100 meeting was more retrospective; it differed significantly from typical research conferences. There were only two formal talks, by Dennis Sullivan and Yair Minsky, based more or less on Lipa's legacy. Informal conversations during the day and the evening banquet gave ample opportunity for attendees to recollect interactions with and describe the lasting influence of Bers.

The Bers 100 celebration and this volume are purely volunteer efforts, and thanks are due to all that participated in the project, most of all to the contributors to this volume, and the referees and readers of their articles. With a non-zero probability that we will ignore important contributors, we single out some for special thanks. First and foremost, we thank Ruth Bers Shapiro and Victor Bers for editing Lipa's autobiographical notes and getting them ready for publication. We relied strongly on the generous advice from Clifford Earle about mathematical history and on the suggestions of Dennis Sullivan about the directions of the entire enterprise. Sergei Gelfand and Christine Thivierge of the AMS were generous in providing assistance in the preparation of this volume. The editors were strongly influenced by Sergei's advice on both the organization and content of this book.

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<sup>3</sup>Under a different title.

Perhaps it is best to end this introduction with a Bers story. In a graduate course he was giving, he made a conjecture which he said was so beautiful that he would buy a case of champagne for anyone who could prove it. A few weeks later, a student came to him with a counterexample. When Bers told this story in one of his lectures, somebody asked “So did you give him a case of champagne?” Bers scowled and said “No, I gave him a Ph.D.”

November 2014

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