Lipmans Bers, A Mathematical Mentor

a panel, January 26, 1984, Joint Mathematics Meetings, Louisville, Kentucky. The talks were given in order of year of Ph.D. Leslie Sibner was unable to attend the actual panel, but her talk is included here.

Welcoming remarks by Linda Keen, moderator, Lehman College, CUNY

We welcome you here today to our panel, "Lipman Bers, A Mathematical Mentor". Lipman Bers will be seventy this spring. Let me tell you how we at AWM come to hold this event. Linda Rothschild suggested to me that because Bers had many women students it would be most appropriate for AWM to sponsor a panel discussion on Bers as a mathematical mentor. Bers had an unusually large number of students, both male and female. To date 47 individuals have obtained Ph.D.'s under Lippa's supervision, and one student is now studying with him. Of these, 16 are women. Undoubtedly, few people can match this record.

Bers' role as a teacher is only a part of his general influence on the mathematical community. He has made important scientific contributions, of course, and continues to be a leader of the professional community. Bers' mathematical interests span several areas of mathematics. His earliest work was in fluid dynamics; in the late fifties Lippa started working in Riemann surfaces and doing his best work. His collaboration with Lars Ahlfors, which began with a paper entitled "Variable Metrics for Teichmüller Spaces" and contains the theorem which Sullivan dubbed the "Measurable Riemann Mapping Theorem", led to a new and very exciting area of mathematics, the theory of Kleinian groups. This area has blossomed, and Lippa has guided many of his students into it. Bers' mathematical activities are matched by his concern that our world be one in which people can live with dignity and freedom. He has been an activist in the world-wide struggle for human rights.

Let me tell you a little of Lipman Bers' story. He was born in Riga, Latvia into a Jewish intellectual family. Concern for a better society led him into politics as a teenager. His political activity got him into trouble with the Latvian government and a warrant was issued for his arrest. Bers escaped from Riga and went to Prague. There he studied at the German University with Karl Loewner and received his Ph.D. in 1937. Prescience and luck were with him again, for he left Prague and went to Paris just before the Munich Pact was signed in 1938. Bers left Paris with his wife (who had left Riga and joined him in Prague) and small daughter just

ahead of the occupying forces. In Lisbon, he was able to obtain a visa to come to the United States.

In the U.S., his first job was at Brown University. Brown at this time hired many refugees, paid them poorly and exploited their talents. Here, Bers had his first 3 students. After the war, Bers moved to Syracuse University where he had 3 more students and his first woman student. After a stay at the Institute for Advanced Study Bers took a position at New York University. At N.Y.U. he had a disproportionately large share of students, 22 over a period of 14 years. It was during this time that the seminal work on Kleinian groups was done, and it provided fertile ground for thesis topics. There were many women students at N.Y.U., perhaps because of its location in a large metropolitan area. Of the 22 N.Y.U. students, 9 were women. In 1964, Bers moved uptown to Columbia. There again he had a relatively large number of students, 18 and 1 still working. Again, Columbia, because of its location, had a relatively large number of women students, and of the 19, 7 were women.

During his career, Bers has received many honors. I will list only a very few of them today. He was elected to the National Academy of Sciences and has been chairman of their Human Rights Committee. He was elected president of the American Mathematical Society and also served as chairman of their Human Rights Committee. He is a member of the American Philosophical Society and the American Academy of Arts and Sciences. He was awarded the Steele prize for his article "Uniformization, Moduli and Kleinian Groups" in the *Bulletin* of the London Mathematical Society.

We cannot begin to talk about all these aspects of Bers' career today. We will, as a group of his students, try to tell you how we perceived him as a teacher and friend. Each of the panelists will tell you something about his or her mathematics and how its development was related to interaction with Lippa. We will try to convey his generosity with his ideas and his warmth and concern for people, be they students, friends or colleagues.

Tilla Klotz Milnor, Rutgers University

This is a case history, a highly personal account of my interactions with Lipman Bers when I was a student at NYU during the mid 50's, and of the influence which Lippa has had on my work.

The first real contact I had with Lipman Bers was in his course in complex variables which I took during my senior year. It would have been hard for Lippa not to notice me. As the course progressed, I became more and more enormously pregnant. It clearly pleased Lippa to have an extra bit of drama in the class. *Will she or won't she arrive this week, even bigger than before?* But the business of the course was mathematics, and Lippa provided the drama there. *Can this week's lecture possibly be as good as last week's?* It always was, of course.

The next year, during my first three months in graduate school, Heinz Hopf visited NYU. Bers found me in the hallway early that semester, and when he heard that I was registered for Hopf's course in classical surface theory, he insisted that I attend as well the seminar which Hopf was to run for faculty and temporary members. You can imagine then my terror when, at the organizational meeting of that seminar, I heard Lippa's booming voice loudly volunteering me to give a talk.

I can't say that I felt singled out by such attention. Lippa has a highly managerial style with students, and for the goodly number of us who didn't know up

from down, it was an incredible boon to receive direction, before we even asked for it.

Later that first year, Lippa urged me to read and explain to him a paper in surface theory which established an important special case of a conjecture due to Carathédory - and thereby an extremely special case of a much larger conjecture in analysis due to Bers' own mentor, Loewner. I found some gaps in that paper's arguments, and by the time I filled in all the resulting holes, I seemed to have a thesis.

That was a bit of an accident, and Lippa wanted me to do better. He suggested that I work on the conformal imbedding problem. (This was before Garsia proved, partly with Rodemich, that every compact Riemann surface can be conformally realized as a surface sitting in 3-space.) I made some feeble attempts to solve that problem, and didn't get very far. But the pattern of my interests was firmly set. Though I work on the differential geometry of immersed surfaces, I have a marked preference for questions and methods which mix geometry with particularly simple complex analysis.

To give some indication of what that means, let me cite two elementary ideas which I have used a lot, both taken from Lippa's course in Riemann surfaces. First is the fact that any positive definite real quadratic form makes an oriented surface into a Riemann surface. Indeed, Lippa spent several lectures proving in ultimate detail that isothermal coordinates "always" exist. When a positive definite real quadratic form has particularly interesting differential geometric properties, then it often pays to work on the Riemann surface it determines. Thus, for example, you can show that the Codazzi–Mainardi equations from classical surface theory reduce to the Cauchy–Riemann equations not just on surfaces in 3-space with constant mean curvature (as Hopf had noted) but on surfaces in 3-space with constant non-zero Gauss curvature as well. You just need to use the second fundamental form, or the second skew-fundamental form to determine conformal structure.

A second idea which I have often used is a sort of backward version of the first. Once conformal structure is set and determined, it can help to normalize the conformal metric so as to give it nice properties. This notion goes back to the uniformization theorem, which guarantees the existence of a conformal metric with constant intrinsic curvature. But other geometric properties can be convenient too. If you are studying harmonically immersed surfaces, for example, there is always a prescribed metric g, and only the conformal class of g matters. If you want to show that harmonically immersed surfaces with positive definite prescribed metric g behave a lot like minimal surfaces (which they do), it helps to normalize g so that the energy function of the immersion is identically equal to 1. If you want to get some handle on the behavior of harmonically immersed surfaces with indefinite prescribed metric g, it helps to normalize g so that area is preserved by the immersion. In both cases, the Codazzi–Mainardi equations can be shown to hold, and because of the normalizations, you get reduction to the Cauchy–Riemann equations, even in the case when the prescribed metric g is indefinite.

But let me get back to less technical matters. I was a student at NYU during the era of Sputnik. Job opportunities were expanding, and there was room in the field for anyone who love mathematics, or just one small part of mathematics. The atmosphere at the Courant Institute was open and friendly. Superimposed on this comfortable backdrop was the simultaneously demanding and encouraging

omnipresence of Lipman Bers. His courses were a model of clarity and insight, and we all shared in the excitement of Lippa's early work on Teichmüller space. Lippa made us feel like members of an extended family, with Mary Bers, his wife, providing unconditional acceptance and approval.

If this all sounds a bit like Camelot, I suppose that is true. But Lippa's uncommon style in dealing with graduate students has produced similar results during far harder times.

Lippa has always had more to give than I could [sic] possible make full use of. For the bounty which I was privileged to share, I am deeply and permanently grateful.

Thank you, Lippa.

Lesley Sibner, Polytechnic Institute of New York

As a graduate student, I did not take the usual route to becoming a student of Lipman Bers. Riemann surface theory was not my forte. Those funny-looking polygons always made me uneasy, and, in fact, they still do. Lippa gave me a problem in partial differential equations instead. It concerned a linear second-order equation in two variables which is much beloved by the Russians, called the Lavrentiev–Bitsadze equation. Solving that problem was the hardest work I have ever done. The entire area of equations of mixed type is still very little understood, and an immense expenditure of effort went into obtaining every small result. However, I really learned how to do hard analysis.

Looking back on it, I see my dissertation as a "warm up" for the next problem suggested to me by Lippa which was then to keep me and my husband busy for the next ten years. Several years after I got my degree, Bob and I flew to an AMS meeting on the same flight with Lippa. He raised the question of proving a "Hodge" theory for compressible fluid flow on a Riemann surface. After all, the actual Hodge theorem for a one-form does describe incompressible fluid flow on a surface. This problem formed the major part of my work thereafter and led me to learn non-linear partial differential equations, the calculus of variations and a fair amount of differential geometry. I had a late start, but I had finally found my field.

When someone asks me who my adviser *was*, I always reply that I *am* a student of Lipman Bers. Being a Bers student is a lifetime experience which does not end when one gets one's degree. Both Mary and Lippa have been extremely close friends all these years. Their generosity, warmth and friendship are things which I value very highly in my life.

Irwin Kra, SUNY, Center at Stonybrook

I had the honor of being Lipman Bers' first Columbia Ph.D. I started graduate work in September, 1963. When I passed the written comprehensive examination in May of 1964, I naturally started to think about areas of specialization. I was vaguely interested in classical analysis, and Columbia in those years offered few choices to would-be analysts. During the summer of 1964 I was at a loss in trying to choose a topic of specialization. When classes resumed in September, Columbia had a new analyst who had moved uptown from NYU and who was giving a course on Riemann surfaces. That course and the one he gave the following year on moduli shaped my mathematical career.

I do not know why Bers moved from Courant to Columbia. From the composition of this panel, one might suspect that he was looking, as part of an affirmative

action scheme, for male graduate students. Whatever the reason, the move was an extremely fortunate one not only for a large group of students at Columbia but also for Bers himself.

So in September 1964, without knowing it and certainly without filling out an application, I became a member of a small mathematics fraternity: the Bers–Ahlfors Mafia—a subset of a larger collection of people working on Riemann surfaces, moduli, and Kleinian groups. This larger group consisted of between 50 and 100 people, and Lipman Bers knew each one both personally and professionally.

I learned quite early that Lippa has a selective memory. He, of course, knew or knew of all the work previously done and currently being done in his fields. However, when it came to students he remembered only their good ideas and bright remarks; he conveniently forgot all their mistakes and the many times they came to him with half-baked and foolish ideas. Further he encourages and applauds independence on the part of his students and is happy when a student or former student proves a new theorem—even when it improves one of his results.

In 1948, Bers showed how an open Riemann surface can be reconstructed from its ring of holomorphic functions. His 1964/65 course, referred to earlier, concluded with a discussion of this theorem and the algebraic structure of the ring of holomorphic functions. Because of the atmosphere at Columbia, I was fascinated by this interplay between algebra and analysis. I asked Bers whether I could choose this topic for my Ph.D. dissertation. He tried to discourage me and told me that his next course on moduli would contain many more promising areas of research. However, we agreed that for a semester I would work on function algebras on Riemann surfaces. Bers relates this incident a little differently. In his recollection, our conversation went approximately like this:

Bers: I am not interested in function algebras any more and besides the area is probably dead.
Kra: Professor Bers, I am not asking you to write a thesis on function algebras. I will do that, I merely want you to guide me to the literature and supervise my efforts.

I, of course, did not have the *chuzpa* for such a reply. But it is quite possible that Lippa's story, when told to chairmen and deans, resulted not only in job offers but also started me on my administrative career. (It is not clear that I should thank Lipman for this.)

Bers, of course, did everything possible to help me in my work on function algebras. Since he was not too interested in the subject, he made sure that Paul Rosenbloom joined the Columbia faculty in 1965/66. Rosenbloom had a long-standing interest in this set of questions and was the possessor of a partially finished manuscript (joint work with Ian Richards) on the characterization of rings of meromorphic functions on open surfaces.

When I obtained, a few months later, an algebraic/topological characterization of the ring of holomorphic functions, I gave Bers an outline of what I hoped would turn out to be a thesis. On a Friday morning, I handed in a list of definitions and theorems without proofs. Bers did not quite believe that my theorems were correct. After a hectic weekend reviewing all the arguments, I presented them to him on Monday morning. That afternoon we met in the elevator and Lippa told me that my arguments looked OK and would be enough for a Ph.D. at NYU—but

at Columbia he was not sure.[1] Two days later, in the Commons Room, he asked where I wanted a job. I assumed that Columbia's standards were not higher than those at NYU and proceeded to type my thesis. But, of course, Bers was right in his initial advice. Function theory on open Riemann surfaces was indeed a dead-end for me. It was the material of his 1965/66 course on moduli that was the foundation of much of my research activities during the subsequent 17–18 years.

It was in that course that I first encountered the names and ideas that would constitute my scientific life: Ahlfors, quasi-conformal mappings, Kleinian groups, Teichmüller spaces, Poincaré series, finiteness theorems, extremal problems. We (his students) were there when Bers put the finishing touches on his area inequalities. The excitement of mathematics was more intoxicating than the subsequent fads that swept Columbia.

For quite a number of years, the field in which we were working was on the fringes of "respectable" mathematics. The "we" consisted in great part of students and adopted students of Lipman Bers. It was and is a group of people who generally supported (mathematically) each other and competed in a mostly gentle manner. It is within this group that I made the closest mathematical and personal friendships. In addition to those on the panel and Lippa, I include my two mathematical brothers Fred Gardiner and Bernie Maskit (the latter is also my colleague at Stonybrook) as well as my mathematical cousin Cliff Earle.

As I mentioned earlier, the move uptown also had some benefits for Lipman Bers. He started using with increasing frequency such words as "cohomology" and "fiber spaces",[2] and his mathematics continued to influence even wider groups of mathematicians. Today Riemann surfaces and Kleinian groups are central to mathematical research. The field is no longer the exclusive domain of a small group, but has several hundred practitioners. One of the reasons for this popularity is the influence of the fundamental papers of Lipman Bers and Lars Ahlfors. In speaking about Bers, the mathematical mentor, one cannot forget about his good friend and mathematical collaborator Lars Ahlfors. The papers of these two mathematicians have a number of things in common. Their papers, of course, attack in elegant manners, central mathematical problems. These papers also point the way to future directions for research; not only for themselves but also for others. Ahlfors and Bers always give generous credit to the accomplishments of other mathematicians. This generosity is part of the cement that helps form the mathematical community of which I am a member.

I never enjoy mathematics more than when telling Lippa about a new theorem. His delight exceeds my own and always renews my enthusiasm for the subject. One of the reasons mathematics is such a pleasure is the presence of mentors such as Lipman Bers and the extended family he has gathered around him; not only students and adopted students are included in this circle. Outstanding mathematicians who have started in quite different areas, for example, Thurston and Sullivan, and outstanding mathematicians who revived entire fields, for example, Ahlfors, are also included. Together, they are breathing new life into an active and healthy old field.

[1] At this point, I thought that I heard a gentle "boo" coming from the direction where Cathleen Morawetz was sitting.

[2] Contrary to rumors, his papers from the NYU period already refer to Eichler cohomology.

Jane Gilman, Rutgers University

I was a Bers student at Columbia. I came to Columbia with the idea that I would work in some area of algebra. Bers was teaching the first year Complex Analysis course that year, and I took it. Bers' approach in that course, and in all his mathematical work, is so conceptual that he makes very deep hard analysis seem as simple and elegant as algebra. So I ended up working in Teichmüller theory.

I was not the only graduate student that year who was influenced by Bers' great skill as a teacher and expositor. There were fifteen other students who entered Columbia with me, and eight of us became his students.

Because Bers knew that I was interested in algebra, he suggested that for a thesis problem I try to prove that the moduli space for punctured Riemann surfaces was a quasi-projective variety. I associated to the punctured surface a two-sheeted covering branched over the "punctures" and much of the algebraic problem was reduced to studying involutions. Although this was not the direction Bers intended, it was the beginning of my interest in conformal automorphisms and algebraic properties of the mapping-class group.

In addition to a solid mathematical training, Bers' students were also taught about mathematical style. Once I had a complete and correct draft of my thesis, Bers spent an afternoon with me completely rewriting the first section. He added a definition here and separated a lemma there. Although the mathematical content was not changed, the new document was more elegant, clearer and more likely to be read. I never write up a paper without thinking about the lessons of that afternoon.

After my thesis Bers continued to exert an influence over me and my work. The type of influence has varied at different stages of my career.

Immediately after my thesis I spent time studying automorphisms of Riemann surfaces and their action on homology. During this time Bers left me alone and urged me to follow my own instincts in choosing problems. He was always encouraging: when someone suggested to him that I should speak in his seminar, he told me that "the masses were clamoring for me to speak".

My work became more closely related to his when I discovered that Nielsen had studied what Bers termed parabolic mapping-classes. It was exciting to be able to say to Bers "did you know that Nielsen had studied parabolic mapping-classes?". Bers encouraged me to explain the connections between Nielsen theory, Thurston's topological classification of the mapping-class group and Bers' subsequent analytic classification.

Recently I have been interested in questions about product relations in the mapping-class group. Again Bers' influence is indirect but pervasive, for it is the context in which he has placed the classification theory which makes these natural questions to ask.

I hope that this personal perspective has served to convey a sense of how and why Bers has such a great influence over everyone working in the field and over the direction in which the field moves. Bers has a great technical ability. He also has a rare ability to conceptualize mathematics so that he gives an individual fact or theorem a significance in and beyond itself. I believe that it is this unique combination of talents that is the source of his great influence.

Jozef Dodziuk, Queens College

. . . I met Lippa in 1970 when I became a graduate student at Columbia. I took some courses with him and learned some mathematics, but more importantly he

helped me to learn what mathematics was and how it was done. For example, Lippa asked me to talk about my thesis in his seminar. I had some doubts since the thesis was not in complex analysis. The gist of what Lippa replied was that it was wrong to label and separate various areas of mathematics. It took me a while to realize how right he was. Ten years later I gave another talk in the same seminar when I reported on some results, unquestionably in complex analysis, which were motivated by and related to the subject of my thesis.

Lippa took great interest in all graduate students at Columbia. I started working with Howard Garland, but Lippa was very helpful in formulating the problem whose solution became my thesis. When Garland left Columbia, Lippa became my official advisor. I was fortunate to be able to continue working with both of them.

My undergraduate education in Poland was highly regimented (courses, assignments, exams, etc.). I had to learn in a hurry that mathematics was not like that, that one did things without being told to do them. Lippa set a fantastic example. His enthusiasm, his wide-ranging interests, the quality and quantity of his mathematics were awesome. Yet, I had a feeling that he treated me and other students as equals. Lippa was very generous with his insights and ideas. At the same time he was very demanding. He made it absolutely clear that the responsibility for my success or failure was mine. This created a great deal of pressure but made it easy to be proud of results of my work, if I had the results.

The friendship with Lippa continued and grew stronger. It has been a privilege and good fortune to know him and to have him as a friend.

Doctoral Students of Lipman Bers

Arbarello, Enrico	Columbia	1973	(Co-advisor: H. Clemens)
Bell, David	Brown	1966	
Berg, Paul	NYU	1953	
Chang, Harold	Syracuse	1951	
Chu, Tienchen	Columbia	1977	
Chuckrow (Steinhardt), Vicki	NYU	1966	
Diaz, Joaquin	Brown	1945	
Dodziuk, Josef K.	Columbia	1973	
Engber, Michael A.	Columbia	1972	
Feinberg, Irwin	NYU	1961	
Fernholz, Robert	Columbia	1967	
Gardiner, Frederick P.	Columbia	1967	
Gilman, Jane	Columbia	1971	
Halpern, Noemi	Columbia	1978	
Jaffe, Sondra	NYU	1962	
Kalme, Charles I.	NYU	1967	
Keen, Linda	NYU	1964	
Kiremidjian, Garo	Columbia	1971	(Co-advisor: M. Kuranishi)
Klotz (Weinstein), Tilla	NYU	1959	
Koppelman, Walter	NYU	1957	
Kra, Irwin	Columbia	1966	
Lee, Chi-yuan	Washington	1955	
Lewis, Jacquelline	NYU	1962	
Linch (Harvey), Michelle	Columbia	1971	

Lytle, Charles	NYU	1960	
Martens, Henrik	NYU	1962	
Maskit, Bernard	NYU	1964	
Mussman (Levy), Dorothy	NYU	1958	
Nagel, Alexander	Columbia	1971	
Nirenberg, Ricardo	NYU	1966	
Olsen, Bruce A.	Columbia	1971	
Parter, Seymour	NYU	1958	
Patterson, David B.	Columbia	1969	
Polonsky, Ivan	NYU	1957	
Protter, Murray	Brown	1946	
Resnicoff, Gita	Columbia	1982	
Riera, Gonzalo G.	Columbia	1977	
Rodlitz (Phillips), Esther	NYU	1960	
Rodríguez, Rubí E.	Columbia	1981	
Russell, Gary L.	Columbia	1976	
Saltzer, Charles	Brown	1949	
Schechter, Martin	NYU	1957	(Co-advisor: L. Nirenberg)
Schechter, Samuel	Syracuse	1952	
Sibner, Lesley	NYU	1964	(Co-advisor: C. Morawetz)
Sibner, Robert	NYU	1962	
Slutskin, Lev	Columbia	1988	
Wason, Judith R.	Columbia	1973	
Weiner (Berger), Marion	NYU	1966	
Wells, Raymond	NYU	1965	
Wetherell, (Ferentz), Elizabeth	Syracuse	1959	
Williams, Eddie R.	Columbia	1971	
Wong, Chak-kuen	Columbia	1970	
Yeh, Fu	Columbia	1972	

According to the Mathematics Genealogy Project, to August 2014 Lipman Bers has 53 students and 369 descendants.