
Preface to the English Translation

Vladimir Arnold was one of the great mathematical minds of the late 20th century. His work was of great significance to the development of many areas of the field. On another level, Russian mathematicians have a strong tradition of writing for, and even directly teaching, younger students interested in mathematics. This work is an example of Arnold's contributions to the genre.

In 2005, Arnold gave lectures at the Dubna summer camp. This camp is an extraordinary gathering of the Russian mathematical community, in which distinguished mathematicians work to support advanced high school and undergraduate students entering the field. The present book is based on notes from these lectures. As the reader will see, Arnold was very connected to the new generation of mathematicians. One can sense the urgency he felt at delivering his thoughts into hands that might take them farther. The reader expecting a formal mathematical exposition will sometimes not find it here.

One might mistake this style of the work as not just urgent, but sloppy. No. The style is well thought out. Arnold's approach to mathematics—and he makes this quite clear in several passages—was fluid and intuitive. He saw mathematics not as a flat plain to

be surveyed, but as a rugged terrain to explore. The most exciting aspect of mathematics, for Arnold, seems to have been a dynamic search for pattern through examination of many special cases. That is, he held a severely Platonic view of the subject, as one that proceeds as if it were an experimental science—hence the title. After this exploratory phase, one can tuck in the ragged edges. Arnold does this in many—but not all—cases, giving us theorems and proofs in the classic manner.

But it is in the chase, in the experimental “phase” of the process of doing mathematics, that Arnold here seems to take the most joy, and offers this joy to a new generation. Mathematical mainstream culture, in which one burns one’s scrap work, discourages this. Few mathematicians—indeed few scientists in any field—open their minds so completely as he has to their students.

Arnold’s style is unforgiving. The reader—even the professional mathematician—will find paragraphs that require hours of thought to unscramble. In some cases, Arnold collapses an argument into a few sentences that might take up several pages in another style of exposition. In other cases, he gives an intuitive argument in place of a rigorous one, leaving the reader to construct the latter. He probably felt that the real work was done on the intuitive level, and that his teaching would be the more effective if he left the tidying up to the student. The reader must have patience with the ellipses of thought and the leaps of reason. They are all part of Arnold’s intent.

These lecture notes were gathered in haste from the field, and we have corrected numerous misprints and small errors in notation. We have given several extensions—in Arnold’s own style—to the work, in “editors’ notes”. At the same time, we have striven to deliver intact the style of the work. Arnold’s mind leaps from peak to peak, connecting disparate areas of mathematics, all (or most) accessible to the student with an advanced high school education. And yet there is a unity to each lecture, a flow from very simple questions to deep intellectual inquiry, and sometimes right to the edge of our knowledge of mathematics.

We hope that we have preserved this coherence, but also the excitement of the work, the sharp, jagged edges and breathtaking jumps that characterize the author's thinking.

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