

Preface to the English Edition

Vladimir Arnold was one of the great mathematical minds of the late 20th century. He did significant work in many areas of the field. On another level, Russian mathematicians have a strong tradition of writing for, and even directly teaching, younger students interested in mathematics. The present volume contains some examples of Arnold's contributions to the genre.

"Continued Fractions" takes a common enrichment topic in high school math and pulls it in directions that only a master of mathematics could envision. While it exemplifies for the student the kind of generalization and abstraction that mathematicians routinely engage in, it does so in a completely non-routine way. The essay also has a powerful lesson for all of us. The author claims to have set out to invent a completely useless (i.e., inapplicable) mathematical construct, yet found that people came to his door asking about it because it was just what they needed for a particular application. Mathematicians, it seems, do more than build a better mousetrap. They seem to invent new creatures to be trapped.

In "Geometry of Complex Numbers, Quaternions, and Spins" the context is physics, yet Arnold artfully extracts the mathematical aspects of the discussion in a way that students can understand long before they master the field of quantum mechanics.

"Euler Groups and Arithmetic of Geometric Progressions" treats a similar enrichment topic, but it is rarely treated with the depth and imagination lavished on it here. Arnold sets it in a mathematical context, bringing to bear numerous tools of the trade and expanding the topic way beyond its usual treatment.

"Problems for Children 5 to 15 Years Old" must be read as a collection of the author's favorite intellectual morsels. Many are not original, but all are worth thinking about, and each requires the solver to think out of his or her box. Dmitry Fuchs, a long-term friend and collaborator of Arnold, provided his solutions to some of the problems. Readers are of course invited to select their own favorites, and construct their own favorite solutions.

In reading these essays, one has the sensation of walking along—sometimes being dragged along—a simple footpath that is found to ascend a

mountain peak, and being shown a vista whose existence one could never suspect from the ground.

Arnold's style of exposition is unforgiving. The reader—even the professional mathematician—will find paragraphs that require hours of thought to unscramble. In some cases, Arnold collapses an argument into a few sentences that might take up several pages in another style of exposition. In other cases, he gives an intuitive argument in place of a rigorous one, leaving the reader to construct the latter. He probably felt that the real work was done on the intuitive level, and that his teaching would be the more effective if he left the tidying up to the student. The reader must have patience with the ellipses of thought and the leaps of reason. They are all part of Arnold's intent.

These notes were often gathered from the field, and we have corrected numerous misprints and small errors in notation. We have also given several extensions—in Arnold's own style—to the work in "Editors' Comments". At the same time, we have striven to deliver intact the style of the essays. Arnold's mind leaps from peak to peak, connecting disparate areas of mathematics, all (or most) accessible to the student with an advanced high school education. And yet there is a unity to each essay, a flow from very simple questions to deep intellectual inquiry, and sometimes right to the edge of our knowledge of mathematics.

We hope that we have preserved this coherence, but also the excitement of the work, the sharp, jagged edges and breathtaking jumps that characterize the author's thinking.

It is our pleasure to acknowledge the contributions of several colleagues to this work. In particular, Sergei Gelfand, at the American Mathematical Society, kept us on track at several key junctures. Paul Zeitz made important contributions to the work of translation. James Fennell sedulously proofread the manuscript and corrected the \TeX files. We would also like to thank the students of the Gradus ad Parnassum math circles at the Courant Institute, who gave us feedback about several sections. Much of this work was supported by a generous grant from the Alfred P. Sloan Foundation.

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