Preface

Who the target audience of this book is

This book is intended for people who are already running a math circle or are thinking about organizing one. It can be used by parents to help their motivated math-loving kids or by elementary school teachers. We also hope that very bright fourth or fifth graders will be able to read this book on their own.

What math circles are

For about a century math circles blossomed in Eastern Europe. Twenty years ago they began to proliferate throughout the United States.

There are so many different flavors and approaches to math circles that it would be impossible to give any definition of what a math circle is. For us the defining common feature of all math circles is that interaction between the students is a significant driving force of the meetings, encouraging kids to learn from each other.

What makes this book special

The main features of this book are the logical sequence of the problems, the description of class reactions, and the leading questions or hints given to kids when they get stuck. This book tries to keep a balance between two goals: inspire readers to invent their own original approaches, while being detailed enough to work as a fallback in case the teacher needs to prepare a lesson on a short notice.

Who we are and how this book came about

Until about 10 years ago math circles were almost exclusively limited to middle and high school students. There was a widespread belief among math circle organizers (including the authors) that math circles do not make a lot of sense until the age of 12.

In Berkeley, the ice was broken in the fall of 2009 when Natasha Rozhkovskaya, while visiting MSRI, gathered a group of about 30 kids, grades 1–3, at UC Berkeley. The program was a huge success, creating an enormous demand for similar programs. The program continued to grow at UC Berkeley after Natasha's departure and then spread to other locations.

This book is based on selected topics we taught in math circles for elementary school students at UC Berkeley, Stanford, Dominican University (Marin County, CA), and the University of Oregon (Eugene).

Our programs enroll over 500 kids each year, with many still remaining on the waiting lists. There is no real selection process and the children are split into two age groups: grades 1–2 and 3–4 (or 2–3 and 4–5 in the Eugene math circle). Weekly sessions are 50–55 minutes long, with 25–30 students (and two instructors) per group. Being closely connected to the University of Oregon, the Eugene Math Circle is an exception: a number of undergraduates work as assistants so that there is approximately one instructor per six students, thus allowing more one-on-one time. In the case of a smaller teacher/student ratio we write very detailed class notes and require the parents to go over them with their kids as a part of the homework. This allows us to have significantly deeper discussions in the class.

How we select our topics

For classes we try to choose topics with deep mathematical context, which are parts of a continuously developing stream of mathematical thought. These topics are just as engaging and entertaining to the children as typical "recreational math" problems, but they can be developed deeper and to more advanced levels.

Our goal is to come up with a theme that will be exciting for first graders and can be taught to fourth graders, and later to middle schoolers, high schoolers, undergrads, graduate students, professors, and so on. There are naturally arising connections between such themes, which provide opportunities even for the young students to see how rich the interplay of math concepts can be.

While Olympiad-style problems are very important tools that attract many kids into math by challenging them, we do not need these tools. Our approach is to keep kids interested by presenting mathematics as a particular way to explore the world. We use Olympiad-style problems only occasionally as warmups and challenges.

As time goes by, we continue to learn how to present more and more advanced topics in our classes. Some of the themes we cover now were not imaginable even a couple of years ago. When immersed in a certain topic, the children comprehend surprisingly advanced material at amazing depth, allowing them to solve very challenging problems.

Since we enjoy coming up with new themes, we have enough to almost never repeat the same topic to the same kids.

How we teach

To hold the kids' attention and to fuel their enthusiasm, we vary the pace and topics by interlacing the main theme with warmup problems, discussion of the homework, and occasionally hands-on projects. We also spice up the presentation of the main topic by calling kids to the board, engaging them in dialogues, and encouraging independent problem solving. Occasionally we give an introductory warmup problem a week or two before starting an in-depth discussion of a new theme to let the material sink in. Warmups also help to cope with late arrivals. We keep a stack of challenging problems for the kids who are ahead of the rest of the class.

It is very important to balance two conflicting requirements: making problem statements unambiguous for the children and at the same time easy to understand. Presenting problems as funny or even silly stories excites the kids and keeps them engaged. Most of our problems have names or characters associated with them; when we later address such problems by these names, the kids easily recall them. We always have a bag of manipulatives (little cubes, coins, sticks, etc.) for the young hands-on learners.

Quite a few kids need to overcome their fear of making a mistake. We help them by turning question-and-answer exchanges into playful dialogues where making a mistake is a part of the game. We tell the kids, "The best mathematicians make a lot of mistakes working on a problem; it is a part of the process. What is important is that they correct them later." Our students even came up with an inside joke: "I made a mistake. I am done with the first step of solving the problem."

Many kids struggle with verbalizing their ideas, even after successfully solving the problem. Moreover, some of them do not see what the point is in explaining their solution. We have to return to this again and again, asking the class to repeat or improve an already presented solution. This introduces students to the basics of precise mathematical discourse.

A few observations

We are often asked how to draw more girls into math circles and keep them involved. In our experience, it happens without any additional effort on our side. The girls typically start stronger and progress very well. More than that, we were told that the percentage of girls in the middle school math circle went up significantly since our elementary program had started.

Another surprising fact is that while we have almost no pre-selection of the children, our retention level is very high. Only a few kids drop out after a semester, and many stay until they grow out of our program, continuing to middle school, and later high-school levels of the math circle, which are run separately (and differently).

In many cases we cover the same material in grades 1–2 (2–3 for Eugene) as in grades 3–4 (4–5 for Eugene), although at a slower pace and in lesser depth. Surprisingly, if we take into account the amount of prior knowledge and stamina for sitting and focusing longer, the difference between the older and the younger groups is not that significant. The younger students digest information slower, and the older students retain it better; nevertheless, at times the younger classes proceed with the same speed as the older ones.

Structure of the book

Each of the first five chapters represents one of the larger themes. Material in Chapter 1 can be either a stand-alone topic or taught as a part of the other themes. The other four themes are mostly independent from each other and comparable in difficulty. From experience we know that all classes are different, so we do not believe it makes sense to estimate how much time a particular topic will take. However, we do not expect that these five themes can take any less than two years to cover.

The remaining chapter and section provide supplementary material. Chapter 6 contains selected warmups and challenging problems. As we have already mentioned, this chapter includes the problems to be given 1–2 weeks before a specific topic is introduced. At the end of the book is a short section containing handouts with biographies of the mathematicians mentioned in the book.

The notes for the teachers are marked by 

. They provide practical insight and suggestions for math circle instructors on organizing classes.

"Math Context" notes give more formal and sometimes more advanced context for the conclusions the students came to while working on the problems.

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