

# Contents

Preface	vii
Chapter 1. Numbers as Geometric Shapes	1
Examples of Figurate Numbers	1
Square Numbers	2
Rectangular Arrangements	7
Triangular Numbers	11
Quick Summations	16
Cubic Numbers	17
Pyramids	20
Chapter 2. Combinatorics	25
Coloring Beads	25
Mumbo Language	26
Ice Cream Cones	27
Nowhere York City	29
The Handshake Problem	32
Sides and Diagonals	33
Same Problems with 10 Objects	33
Apples, Oranges, and More	34
Problems about Numbers	36
Harder Problems	37
Chapter 3. Fibonacci Numbers	41
Building Strips with Squares and Dominoes	41
Parking Problems	44
Counting Routes	48
Fibonacci Sequence in Nature	49
Extension to the Left	51
Even/Odd Pattern	52
Divisibility by 3	55
Sum of the First $n$ Consecutive Fibonacci Numbers	59
Fibonacci Rectangles and Fibonacci Spiral	61
Honeybees' Ancestral Tree	64

Chapter 4. Pascal's Triangle	69
Paths in Mouseville	69
Hockey Stick Pattern	74
Diagonals in Pascal's Triangle	75
Rows in Pascal's Triangle	77
Extending Pascal's Triangle	79
Fibonacci Numbers in Pascal's Triangle	83
Sierpinski Triangle	84
Counting Odd and Even Numbers in Pascal's Triangle	89
Pascal's Triangle Modulo 3	94
Chapter 5. Area	99
Playing with Squares	99
Areas of Similar Shapes	103
SAME SHAPE SAME SIZE	107
Rotation by a Right Angle	109
Area of a Tilted Square	114
Pythagorean Theorem	120
Area of a Parallelogram and Area of a Triangle	126
Pick's Formula	129
Chapter 6. Selected Warmup and Challenging Problems	135
Handouts	163
Bibliography	169
Index	171