

Preface

This book is about enjoying mathematics, about using the mind as an athlete uses the body in a joyful game. Its inspiration is the Julia Robinson Mathematics Festival, an organization that supports non-competitive after-school mathematical activities.

The activities in this book are just a few of the many that the Julia Robinson Mathematics Festival has developed over the years. The problems have a low threshold, but their ceiling is high. A game or puzzle intrigues, then stimulates thought, that then becomes serious mathematics before the reader is aware of the transition. A gentle on-ramp leads to an exhilarating ride through fantastic mental landscapes.

Almost anyone, at any level, can begin to engage in the sequences of problems in each chapter. Most of the chapter problems begin with situations that require no more than simple arithmetic or intuitive concepts of symmetry—and sometimes not even that. On a deeper level, the beginning problems require intuitive logic, but not the notion of a strategy for play or algorithm for solution. These more sophisticated ideas emerge as the student works the puzzle, plays the game, or solves the problems that come later. As soon as solvers can understand the statement of the first problem in a chapter, they can engage in the solution, progressing to the next step whenever they are ready. And if students never get to the next step, they will have learned something significant, just from understanding the difficulties of the situation.

A Julia Robinson Mathematics Festival is a social occasion to learn mathematics. In a large open space, tables are provided on which a variety of problem materials are made available. (Each chapter in this book is a sample of the materials at a single Festival table.) Each table has a table leader or facilitator, who manages the activity—but does not teach in the traditional sense. Students are free to wander among the tables, choosing for themselves an activity to engage in and the amount of time they spend. Informal social groups form around the activities. Students bring old friends and make new friends. Facilitators observe and support, but intervene only minimally.



For those planning a Julia Robinson Mathematics Festival, we have included hints for organization, marked with a Julia Robinson logo in the margin.

Our activities have been selected and presented based on feedback from previous Festivals. A good problem is one that attracts students, gets them interacting, and holds their attention for at least half an hour. The attractive and accessible nature of the problems in this book are a result of observing student engagement at a Festival, followed by a careful honing of the materials to improve them for the next Festival.

We have written this book for a wide audience. We specifically target facilitators at Julia Robinson Mathematics Festivals, or leaders of after-school math circles. However, the book can also be used in more formal situations, including whole class activities where the teacher must give more structure to the activity. We have chosen problems that can be used with or without the structure of a formal classroom. The book can also be used for self-study by a student alone.

We have departed somewhat from the usual presentation of problems and solutions. The focus of the book is on mathematics and teaching. So we have put the activity guides first. The problem statements, which often come first in contest-problem books, are given in Part II. Some readers may want to try the problems themselves, only later looking at the teaching notes. And students working alone should certainly look at the problems first. These readers can simply turn to the activities section at the start of their reading. In fact, there is no reason to begin a reading of the book with Chapter I. Each problem starts the exploration afresh.

The activity guide notes are more than just solutions. They are essays about the mathematics involved, about how the first and simplest steps mimic or foreshadow bolder progress towards deeper results. They also contain teaching notes gleaned from experience with students working these problems. They offer hints for listening to the students and point out the typical (and sometimes atypical) pathways that students find in working the problem.

The historical notes are not meant to be comprehensive. Rather, they are there to entice the reader to look into the people and cultures behind the mathematics. We hope that including such notes in problem books will become standard practice.

The best leaders are followers—followers of the minds of the students, looking at where they are, anticipating their next thoughts, and offering the minimal support necessary for the student to achieve the next insight. Hence, it is important that these notes be used appropriately. They are not meant as lesson plans in the traditional sense. Rather, they are meant as signposts to the user of what might lie ahead and which pathway to take.

So, for example, we sometimes offer a suggestion for a hint to the student, often in the form of a question. It is important that the teacher or leader use these hints sparingly, giving as little support as possible so that

students progress independently in their work. A hint given too early deprives the student of discovery, a hint given too late risks frustration. Timing is everything. But timing cannot be put between the pages of a book. This book—like any teaching guide—is only as good as the leader who uses it. We hope readers will use it wisely.

These caveats are not meant to restrain the reader's delight in the book. However you use it, in whatever order you read the materials, we wish you joy in reading. We have enjoyed putting this material together and have learned a lot in the process. We hope you will learn something as well. And have fun while learning.

—Mark Saul and Alice Peters